Autoregressive Spectral Analysis and Model Order Selection Criteria for EEG Signals

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Abstract: The advantages of autoregressive (AR) modelling over the classical Fourier Transform methods have been centre staged in the recent years. But a problem with AR method lies with the appropriate model order selection. In this paper, we address this problem by studying the performance of three different types of order selection criteria for AR models to represent electroencephalogram signals. We perform this by extracting EEG signals for different mental tasks and obtaining the appropriate model order given by the different criteria. From this, we derive the spectral density function. Using the spectral values, we train a neural network and classify the tasks into their respective categories. In this way, we show the difference in the performance level of the different model order selection criteria for EEG signals.

Keywords
EEG, Autoregressive Spectral Analysis, Neural Network, Final Prediction Error, Akaike's Information Criterion, Reflection Coefficient

I. INTRODUCTION

Linear autoregressive (AR) models have a broad spectrum of applications ranging from identification, prediction and control of dynamical systems. But a problem with AR method lies with the appropriate model order selection. As one can surmise, the AR model can be any order as desired. However, it should be as accurate as possible. Our intuition tells us that a model order, which is too small will not represent the properties of the signal, whereas a model which is too high will also represent noise and inaccuracies and thus, will not be a reliable representation of the true signal. Therefore, methods that will determine the appropriate model order must be used. Some methods can be ascertained from the nature of the modelling process and some of the methods depend on the similar concepts that are used in regression analysis [6].

For this paper, we study the performance of three different types of order selection criteria for AR models to represent electroencephalogram (EEG) signals i.e. Final Prediction Error, Akaike's Information Criterion and Reflection Coefficient. We perform this study by extracting EEG signals for different mental tasks and obtaining the appropriate model order given by the .different criteria. From this, we derive the spectral density function. Using the spectral values, we train a neural network and classify the tasks into their respective categories. In this way, we show the difference in the performance level of the different model order selection criteria for EEG signals.

II. AUTOREGRESSIVE MODELLING

The EEG signals are represented by AR models of order p, which are given by

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + e(n) \tag{1}$$

where p is the model order, $x(n)$ is the data of the signal at sampled point n, a_k are the AR coefficients and $e(n)$ represents the error term independent of past samples. The obtain a_k, we can solve the Yule-Walker equations directly using conventional linear equation solution like Gaussian elimination but a shortcoming of this approach lies in its huge computational time. Thus, recursive algorithms have been developed which are based on the concept of estimating the parameters of a model of order p from the parameters of a model order p-1. One of these methods is known as the Levinson–Durbin algorithm [6], which is the method used in this paper.

The first two model order selection criteria are based upon concepts in mathematical statistics and are pioneered by Akaike [1]. The first of these is the Final Prediction Error (FPE). Using this method, the model order is chosen which minimises the following function

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$$FPE = s_p^2 \frac{N+p+1}{N-p-1} \quad (2)$$

where p is the model order, N is the number of data points, s_p^2 is the total squared error divided by N and is given by

$$s_p^2 = \frac{1}{N} \sum_{p}^{N-1} e^2(n) \quad (3)$$

The fractional portion of FPE increases with p and as such represent the inaccuracies in estimating the AR parameters.

The other criteria is called the Akaike's Information Criterion (AIC) and the model order is selected which minimises the following function

$$AIC = N \ln s_p^2 + 2p \quad (4)$$

The term $2p$ represents the penalty for higher orders.

Another concept used to select the model order is of the idea of partial correlation and this is sometimes known as refection coefficient. It is designated by $\pi_p = a_p$, the last coefficient of the AR model order p. It is a measure of the amount of correlation at lag p not accounted for by a $(p-1)$ order model. In this paper, we use a 95% confidence level and the order k is selected if $p \geq k$, where 95% of π_p values are less than $\pm 1.96/\sqrt{N}$ [6] [1].

Once the appropriate model order is chosen using these criteria, we obtain the AR parameters by using the Levinson-Durbin algorithm. Using these AR coefficients, the power spectral density (PSD) function can be obtained by using the equation

$$S(f) = \frac{s_p^2 T}{\left| \sum_{k=0}^{p} a_k e^{-i2\pi fkT} \right|^2} \quad (5)$$

where $S(f)$ represents the power spectral density function and T is the sampling period. Figure 1 shows an example of an AIC plot for an EEG segment. From this figure, it can be seen that model order 3 minimizes the AIC function and as such is selected to represent this particular EEG segment. Figure 2 shows the PSD values of the EEG signal obtained by using Equation (6).

III. EXPERIMENTAL STUDY

The subjects are seated in an Industrial Acoustics Company sound controlled booth with dim lighting and noiseless fans for ventilation. An Electro-Cap elastic electrode cap was used to record EEG signals from positions C3, C4, P3, P4, O1 and O2, defined by the 10-20 system of electrode placement [2]. The electrodes are connected through a bank of Grass 7P511 amplifiers and bandpass filtered from 0.1--100 Hz. The data was sampled at 250 Hz with a Lab Master 12 bit A/D converter mounted in an IBM-AT computer.

For this paper, the data from four subjects performing two different mental tasks are analysed.

- Math task, for which the subjects are given nontrivial multiplication problems, such as 72 times 38, and are asked to solve them without vocalizing or making any other physical movements
- Geometric figure rotation, for which the subjects were asked to visualize a particular three-dimensional block figure being rotated about an axis

As a control, an experiment is also conducted with a 15th AR model order for the EEG signals.

In the experimental study, we fixed the minimum order as 3 and the maximum order as 15. The lower limit is to avoid the PSD from being too flat when an extremely small model order is selected by any of the criteria. The higher limit will avoid the occurrence of peaks caused by spurious signals in the PSD function.

The EEG signals are recorded for a period of 10 seconds. With a 250 Hz sampling rate, each 10 second trial produces 2,500 samples per channel. Overall, there are 8 different EEG files (2 tasks x 4 subjects). Each EEG signal is segmented with a half-second window, i.e. for a length of 125 points giving 20 patterns for each file with a total of 160 patterns for the experimental study. For all the experiments, 50% of available patterns are used for training while the rest 50% are for testing.

Since we are interested in classifying EEG signals, we only extract PSD values in the range of δ, θ, α and β i.e from 0 to 30 Hz. Next, a multilayer perceptron (MLP) neural network trained by the backpropagation algorithm [5] is used with these PSD values for classifying different mental tasks. The neural network is trained until the average error converges to the limit of 0.005. The network's size is chosen to be 180:50:2 i.e. 180 input nodes, 50 hidden nodes and 2 output nodes for the experiments involving different single subjects while the combined subjects experimentation used a 180:100:2 network size. The higher number of hidden units for the latter case is to accommodate the larger variation in inputs caused by different users. The input nodes consist of 30 PSD values for each channel and since we are interested in classifying only two tasks, we need only 2 nodes in the output layer. The performance of the classifier is used to show the difference in the ability of the different model order

[1] Some authors have used a rounded figure of $\pm 2.0/\sqrt{N}$.

selection criteria. Figure 3 shows the neural network model used in this study.

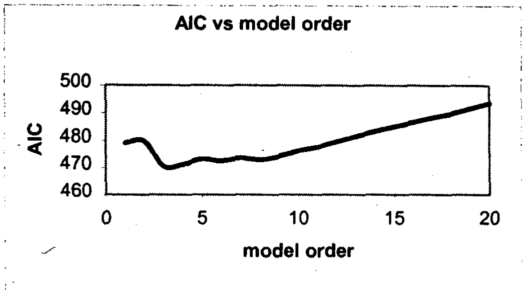

Figure 1: An example of AIC function for an EEG segment

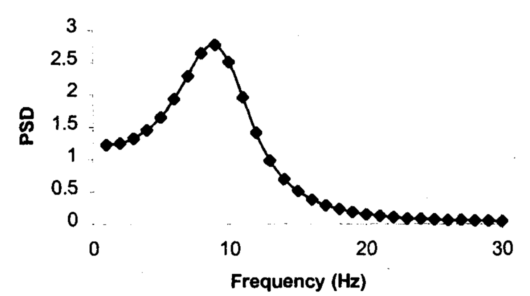

Figure 2: PSD for the EEG segment obtained by using model order 3 given by AIC in Figure 1

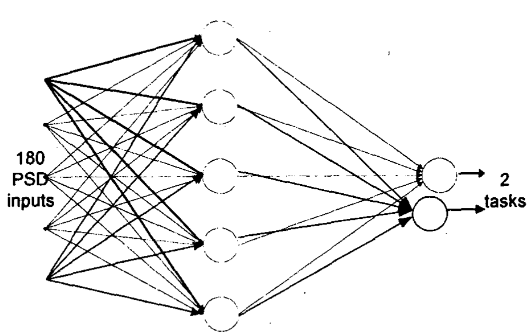

Figure 3: Neural network model as used in this study

IV. RESULTS

Table 1 shows the results of the experimental study for different number of subjects. In general, we can see that the performance (irrespective of the model order selection criteria) varies between different subjects with subject 3 performing the best and subject 4 being the worst. This shows that some users require more training before a successful system could be implemented. But overall, we can see that the system could differentiate the mental tasks to a good accuracy using EEG signals only for each subject.

It can also be seen that the performance of FPE and AIC are about the same since both these criteria give the same model order for most of the EEG segments. As such, either method can be chosen as appropriate to the situation. The method using reflection coefficient gives much lower orders than FPE and AIC, therefore being computationally cheaper. More importantly, it also gives the best performance in most of the cases. Therefore, based on the results, it is faster and better to be used for classifying EEG signals with AR spectral modelling. Performance of a fixed model order 15 for all the EEG signals gives average performance only except for the case of 4 users. This could be due to the noisy spurious peaks present in the PSD caused by high model order. Furthermore, it is computationally too expensive to be used on practical systems.

Table 1: FA classification for different users

Criteria	User			
	1st	2nd	3rd	4th
AIC	90	100	95	70
FPE	90	95	95	75
RC	95	90	100	70
15th order	65	75	95	70

Table 2: FA classification for different number of users

Criteria	Number of users			
	1 user	2 users	3 users	4 users
AIC	90	72.5	70	65.6
FPE	90	70	70	65.6
RC	95	72.5	72.5	68.8
15th order	65	70	66.7	73.1

V. CONCLUSION

In this paper, we have studied autoregressive-modelling techniques for EEG spectral analysis. We have analysed three different AR model order selection criteria namely Final Prediction Error, Akaike's Information Criterion and Reflection Coefficient. We have studied their performance on neural network classification of two different mental tasks. We have also experimented with a 15^{th} order AR model as a comparison. Our results show that not only Reflection Coefficient gives good performance but the method also chooses a lower optimum order (in most cases) as compared to the other model order selection criteria, thereby being computationally cheaper. We also show that it is possible to differentiate between mental tasks to a high degree of accuracy using EEG signals for each subject.

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