

New Invariant Moments for Non-Uniformly Scaled Images

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Abstract

The usual regular moment functions are only invariant to image translation, rotation and uniform scaling. These moment invariants are not invariant when an image is scaled non-uniformly in x and y axes directions. This paper addresses this problem by presenting a new technique to obtain moments that are invariant to non-uniform scaling. However, this technique produces a set of features that are only invariant to translation and uniform/non-uniform scaling. To obtain invariance to rotation, moments are calculated with respect to the x-y axis of the image. To perform this, a neural network is used to estimate the angle of rotation from the x-y axis and the image is unrotated to the x-y axis. Consequently, we are able to obtain features that are invariant to translation, rotation and uniform/non-uniform scaling. Mathematical background behind the development and invariance of the new moments are presented. The results of experimental studies using English alphabets and Arabic numerals scaled uniformly/non-uniformly, rotated and translated are discussed to further verify the validity of the new moments.

Keywords: non-uniform scaling, regular moments, rotation, principal axis, tilt angle, neural network.

1.0 Introduction

The capability of recognising and classifying patterns is one of the most fundamental characteristics of human intelligence. It plays a key role in perception as well as at the various levels of cognition. High-level image analysis system involves pattern analysis where the automatic recognition of an object in a scene regardless of its position, size and

orientation is an important problem. A number of techniques have been developed to derive features from an image, which are invariant under translation, scale change and rotation [1], [2], [3]. Shape normalisation for handwritten characters have been developed by the authors in [4], [5]. In particular, the invariant properties of regular moment functions have attracted many users to utilise them as pattern features in object recognition, pattern classification and scene matching [6], [7]. Hu [1] first published his classic paper on pattern recognition in 1962 by deriving a set of regular moment invariants based on combinations of regular moments using algebraic invariants. He derived a set of invariant moments, which has the desirable properties of being invariant under image translation, scaling and rotation. Besides Hu, Bamieh and De Figueiredo [2] derived another set of moment invariants in which the main characteristic of the result is that the feature vector size is much lower than any other known invariants, which makes them computationally cheaper. These regular moment functions are invariant to changes in scale, shift and rotation.

In this paper, we show that these moment invariants are not invariant when an image is scaled non-uniformly. Next, we address this problem by presenting a new technique to obtain moments invariant to non-uniform scaling. This technique uses a derived equation that gives the x-y axis scale factor of the image. However, this technique produces a set of features that are only invariant to translation and uniform/non-uniform scaling. They are not invariant to rotation. To make them invariant to rotation and to be able to use the x-y axis scale factor equation, moments are calculated with respect to the x-y axis of the image. To perform this, the angle of rotation from the x-y axis must be known. The authors in [1], [8], [9], [10] have suggested the use of second order moments to determine the angle of rotation from the principal axis. But this method in itself cannot be used to

achieve invariance to rotation for images scaled non-uniformly since this rotational angle will be inclusive of the tilt angle. The tilt angle measures the angle difference between the x-y axis and the principal axis and varies for different non-uniform scale factors. Therefore, to correctly determine the amount of rotation from the x-y axis, the tilt angle for the particular scaled and/or rotated image must be known.

To solve this problem, a Multilayer Perceptron (MLP) neural network is trained using the Back-propagation (BP) learning algorithm [11] to estimate the tilt angle of the image and from this, the amount of rotation from the x-y axis for the image can be determined. The neural network is presented with mass normalised moments up to the third order of some of the non-uniformly scaled and rotated images and is tested with the remaining images. The inputs are limited to the third order to reduce the effects of noise. The target value of the neural net is the tilt angle.

With the predicted tilt angle by the neural network, the image is unrotated (not pixel by pixel but by using the relationship between moments and rotation shown in Section V) to make them invariant to rotation. Once invariance to rotation is accomplished by computing the moments in the x-y axis, invariance to uniform/non-uniform scaling is obtained by using the x-y axis scale factor equation. Since the basic computation of the new moments involve the use of central moments, the new moments are automatically invariant to changes in image position. Consequently, we are able to obtain features that are invariant to translation, rotation and uniform/non-uniform scaling.

Mathematical background behind the development and invariance of the new moments are presented. The results of experimental studies using English alphabets and Arabic

numerals are also discussed to verify the practical validity of the new moments. The images used in the experiments are scaled non-uniformly in the x and y axes directions and added with transformations of rotation and translation type. Although gray scale images are commonly encountered, we have chosen to experiment with binary images in this paper. This is since any gray level image can easily be converted to binary images using global thresholding and also since calculations using binary image are computationally cheaper than gray level images.

The basic theory of moments is discussed in Section II. The problem of non-uniformly scaled images is described in Section III. The proposed new moments and its derivation are proposed in Section IV. This section also includes an experiment with English alphabets, which verifies the invariance of the new moments to non-uniform scaling and translation. Tilt angle and the rotational property for moments of non-uniformly scaled images are presented in Section V. The use of a neural network to obtain the tilt angle is covered in Section VI. In our second experimental study in Section VII, we show the improvement in using the new moments derived in this paper as compared to the usual regular moment functions for Arabic numerals, which are rotated, non-uniformly scaled and translated. This section also verifies the ability of the neural network to accurately predict the tilt angle. Section VIII provides the conclusion.

2.0 Basic Theory of Regular Moment Functions

If an image can be thought of as a two dimensional density function, then the usual regular moment definition is given by:

$$m_{pq} = \int_{x_1}^{x_2} \int_{y_1}^{y_2} x^p y^q f(x, y) dx dy \quad \text{for } p, q = 0, 1, 2, \dots \quad (1)$$

where we have assumed for simplicity the region of interest to be defined as

$$x_1 < x < x_2 \quad \text{and} \quad y_1 < y < y_2 \quad (2)$$

and with $p, q \in \mathbb{N}_0$ as order indices, (x, y) are Cartesian co-ordinates, f is a non-negative intensity function with bounded and compact support so that integration within the available image plane is sufficient to gather all the signal information. In this paper we limit ourselves to binary images where $f(x, y)$ can be either 1 or 0.

To make these moments invariant to translation, one can define central moments as

$$\mu_{pq} = \int_{y_1}^{y_2} \int_{x_1}^{x_2} (x - \bar{x})^p (y - \bar{y})^q f(x, y) dx dy \quad (3)$$

where \bar{x} and \bar{y} are the coordinates of the centroid of the image given by

$$\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}} \quad (4)$$

These moments are made invariant to scale change as proposed in [1] by normalising the image intensity to unity

$$\eta_{pq} = \frac{\mu_{pq}}{(\mu_{00})^{(p+q+2)/2}} \text{ for } p+q=2, 3, \dots \quad (5)$$

Rotation invariance can be achieved by combining the moments based on the theory of algebraic invariance as shown by Hu [1]. Rotation invariance can also be achieved by knowing the angle of rotation from any one of the principal axes and using the relationship between the rotated image and its original form [1]. However, the latter method by itself cannot be used for images that are non-uniformly scaled because of the invariance of the tilt angle with the non-uniform scale change. At any rate, both of these methods use a function of the mass to normalise the image and as such, they are not invariant to non-uniform scaling. This is caused by the different scaling factors which are not reflected in the calculation of the image mass.

3.0 The Problem of Non-Uniformly Scaled Images

The usual regular moment functions like Hu Moment Invariants (HMI) [1], Principal Axis Moment Invariants (PAMI) [1] and Bamieh Moment Invariants (BMI) [2] are invariant to changes in shift, rotation and uniform scaling. Authors in [1], [2] have shown that these moments remain invariant when the scale change in the x and y axes directions are uniform and the derived features have been used for many applications [6], [7], [8], [13]. In some of the applications the scale change in the x and y axes directions is not uniform. This may be due to the digital nature of the imagery caused by undersampling and digitising effects. Another possibility is that the image itself in comparison with the standard image has non-uniform scale change in the x and y axes directions. So, when the image is an elongated or compressed version of the original image, as illustrated in Figure 1(a) and (b), then these moments do not remain invariant.

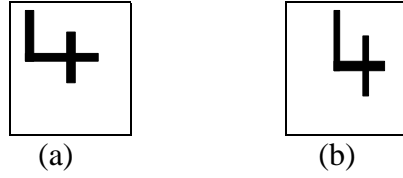


Figure 1(a): Original image 4 and (b): Image 4 with translation and non-uniform scaling

Usual moment invariants like HMI, PAMI and BMI use a function of the mass to obtain invariance to uniform scaling. For images that are scaled non-uniformly, these moments will not hold to invariance anymore since this mass normalisation technique does not take into account the different scale factors of the image. Table 1 shows the results of using HMI for some of the non-uniformly scaled English alphabets. HMI has 7 moment features up to the third order [1]. From this table, it can be seen that these moments are not invariant to non-uniform scaling.

Table 1: HMI and BMI moments for non-uniformly scaled alphabets

Image	Scale factor	HMI_1	HMI_2	HMI_3	HMI_4	HMI_5	HMI_6	HMI_7
P	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	2.87e-1	9.26e-3	7.47e-3	4.70e-4	6.62e-7	3.03e-5	-5.80e-7
P	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	3.27e-1	3.38e-2	1.00e-2	2.00e-3	8.95e-6	3.57e-4	5.35e-7
Q	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	3.32e-1	2.20e-3	7.73e-4	8.96e-5	-2.24e-9	3.50e-6	-2.35e-8
Q	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	3.70e-1	2.89e-2	1.08e-3	3.42e-4	1.92e-7	5.04e-5	-7.96e-8
R	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	2.82e-1	1.13e-3	3.51e-4	3.04e-4	-2.60e-8	9.70e-6	9.56e-8
R	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	3.18e-1	2.30e-2	1.13e-4	4.79e-4	1.59e-10	6.74e-5	1.12e-7
S	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	2.96e-1	1.03e-2	1.96e-4	6.98e-6	2.32e-10	-4.95e-7	1.13e-10
S	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	3.62e-1	5.36e-2	2.13e-4	8.30e-6	-3.40e-10	-1.11e-6	-8.00e-11

Furthermore, PAMI has an additional problem achieving rotational invariance caused by images that are scaled non-uniformly because the total angle of rotation from any one of the principal axes varies with non-uniform scale change for a particular image. This is due to the fact that the tilt angle, which is the angle difference between the principal axis and x-y axis, varies with non-uniform scaling although the amount of rotation is independent to this type of scaling. So for images that are scaled non-uniformly, the method of using PAMI to obtain representative features would pose twofold difficulty of rotation and scaling.

4.0 New Regular Moment Invariants for Non-Uniformly Scaled Images

After some algebraic manipulation of (3), the central moments for the image shown in Figure 1(a) can be expressed in continuous form as

$$\mu_{pq} = \int_{y_1}^{y_2} \int_{x_1}^{x_2} \left(x - \frac{1}{2}(x_1 + x_2) \right)^p \left(y - \frac{1}{2}(y_1 + y_2) \right)^q f(x, y) dx dy \quad (6)$$

where our region of interest is defined as earlier: $x_1 < x < x_2$ and $y_1 < y < y_2$.

Using the binomial expansion, (6) can be expressed as

$$\mu_{pq} = \left[\frac{(x_2 - x_1)^{p+1}}{p+1} + \dots + C_p^p (x_2 - x_1) \left(-\frac{1}{2}(x_1 + x_2) \right)^p \right] \times \left[\frac{(y_2 - y_1)^{q+1}}{q+1} + \dots + C_q^q (y_2 - y_1) \left(-\frac{1}{2}(y_1 + y_2) \right)^q \right] \quad (7)$$

Now, consider the image scaled non-uniformly in Figure 1(b). Let us assume that the expansion in the x and y axes directions to be α and β respectively. The central moments, $\tilde{\mu}_{pq}$ for this non-uniformly scaled image can be defined as

$$\tilde{\mu}_{pq} = \left[\frac{\alpha^{p+1}(x_2 - x_1)^{p+1}}{p+1} + \dots + C_p^p (x_2 - x_1) \alpha \frac{1}{2} (\alpha x_1 + \alpha x_2)^p \right] \times \left[\beta^{q+1} \frac{(y_2 - y_1)^{q+1}}{q+1} + \dots + C_q^q (y_2 - y_1) \beta \frac{1}{2} (\beta y_2 + \beta y_1)^q \right] \quad (8)$$

If now $\tilde{\eta}_{pq}$ is computed using

$$\tilde{\eta}_{pq} = \frac{\tilde{\mu}_{pq}}{(\tilde{\mu}_{00})^{(p+q+2)/2}} \quad \text{for } p+q=2, 3, \dots \quad (9)$$

for this non-uniformly scaled image and expressing it in terms of the original image, η_{pq} calculated from (5) for Figure 1(a), we get

$$\tilde{\eta}_{pq} = \left(\frac{\beta}{\alpha} \right)^{\frac{q-p}{2}} \eta_{pq} \quad (10)$$

where the sign ‘ \sim ’ denotes the moments formed for the image scaled non-uniformly. It is evident from (10) that when $\alpha = \beta$ i.e. when uniform scaling is applied, $\tilde{\eta}_{pq}$ is the same as η_{pq} , thus giving us moments that are invariant to uniform or equal scaling. The authors [6], [7], [9], [10] have all used $\alpha = \beta$ in solving their problems. In [12] the authors have

proposed a technique to make it invariant when $\alpha \neq \beta$. However, only 2 out of 4 features proposed in their paper are invariant to non-uniform scaling. Two invariant features are insufficient for successful representation especially under noisy environments. In this paper, all orders of the new moments are invariant to non-uniform scaling.

If we could obtain the scale ratio or the scale factor, β/α , then moment-invariants can be formed even when $\alpha \neq \beta$. This scale factor can be obtained from the computed moments using a derived equation, which is shown next. But these moments must be calculated with respect to the x-y axis to obtain the correct scale factor since the non-uniform scaling is present in this axis.

The general relationship between the scaled and original image is given by

$$\tilde{\mu}_{pq} = \alpha^{p+1} \beta^{q+1} \mu_{pq} \quad (11)$$

where both $\tilde{\mu}_{pq}$ and μ_{pq} are defined on the same axis. Using the relationship when $p+q=2$, we get the following scale ratio, β/α .

$$\frac{\beta}{\alpha} = \sqrt{\frac{\tilde{\mu}_{02}}{\tilde{\mu}_{20}}} / \sqrt{\frac{\mu_{02}}{\mu_{20}}} \quad (12)$$

This scale ratio β/α relates the scale factor of the scaled image represented by $\sqrt{\tilde{\mu}_{02}/\tilde{\mu}_{20}}$ to the scale factor of a reference image represented by $\sqrt{\mu_{02}/\mu_{20}}$. In this paper, a value of 1.0 is used for this reference image scale factor, $\sqrt{\mu_{02}/\mu_{20}}$ since by doing so will simplify

the computation. The only disadvantage of using this method is that the values of $\tilde{\eta}_{20}$ and $\tilde{\eta}_{02}$ will be the same but this is not really a loss as compared to PAMI features where $\tilde{\eta}_{11}$ is zero, which means that the number of available moment features are same. We can derive other scale factor equations using higher orders but we have chosen a second order moment equation since it is less sensitive to noise. From (10) and (12), we get moments

$$\eta_{pq} = \left(\frac{\tilde{\mu}_{02}}{\tilde{\mu}_{20}} \right)^{\frac{p-q}{4}} \tilde{\eta}_{pq} \quad (13)$$

that are invariant to translation and uniform/non-uniform scaling. Table 2 shows the results of using these new moments for the non-uniformly scaled alphabets shown in Table 1. From this table, we can conclude that the new proposed moments are invariant to non-uniform scaling.

Table 2: New proposed moments for non-uniformly scaled alphabets

Image	Scale factor	η_{20}	η_{11}	η_{21}	η_{12}	η_{30}	η_{03}
P	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	1.42e-1	4.41e-2	2.54e-3	-1.97e-2	2.15e-2	-1.94e-2
P	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	1.42e-1	4.41e-2	2.54e-3	-1.97e-2	2.15e-2	-1.94e-2
Q	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	1.66e-1	-1.98e-2	-1.32e-3	5.25e-3	-8.34e-3	9.40e-3
Q	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	1.66e-1	-1.98e-2	-1.32e-3	5.25e-3	-8.34e-3	9.40e-3
R	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	1.40e-1	-6.32e-3	-9.11e-3	2.55e-3	3.83e-3	-6.57e-3
R	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	1.40e-1	-6.32e-3	-9.11e-3	2.55e-3	3.83e-3	-6.57e-3
S	$\alpha=1.0 \beta=1.0$ $\beta/\alpha=1.0$	1.39e-1	-1.17e-2	-4.18e-3	-1.69e-3	-4.78e-4	1.31e-3
S	$\alpha=1.0 \beta=1.5$ $\beta/\alpha=1.5$	1.39e-1	-1.17e-2	-4.18e-3	-1.69e-3	-4.78e-4	1.31e-3

5.0 Rotational Property of Non-Uniformly Scaled Images

In our previous discussion, we have shown that the new proposed moments are invariant to non-uniform scaling but these moments are not invariant to rotation. We have also discussed the fact that the scale factor equation must be computed from moments defined in the x-y axis. To solve both these problems, we unrotate the image to the x-y axis. In order to do this, we need to compute the angle of rotation

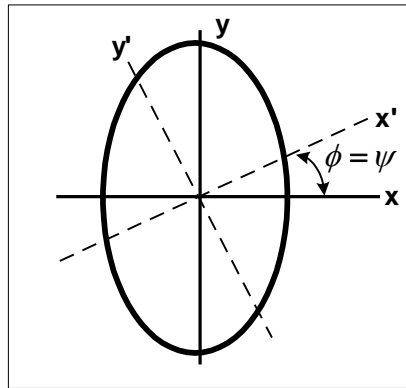
$$\phi = \frac{1}{2} \tan^{-1} \frac{2\mu'_{11}}{\mu'_{20} - \mu'_{02}} \quad (14)$$

where ϕ represents the angle of rotation from the principal axis [1]. Using added restrictions, such as $\mu'_{20} > \mu'_{02}$ and $\mu'_{30} > 0$, ϕ can be determined uniquely. The added restrictions are necessary to define the angle ϕ from 0° to 360° since (14) gives ϕ in the range of -45° to 45° only. The symbol prime denotes moments calculated for rotated images. Many authors who have worked with regular moments have suggested the use of this principal axis to obtain invariance to rotation [1], [9], [10].

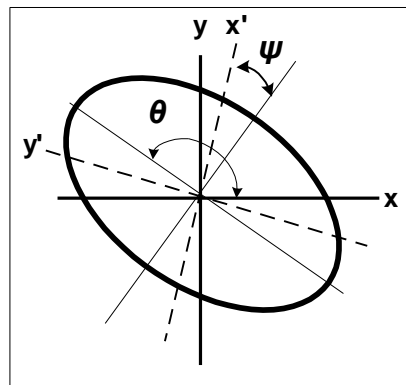
But the problem with the angle of rotation obtained using (14) is that it represents the amount of rotation from the principal axis and not the x-y axis, which is actually what we would like to obtain. To find a solution to this problem, let us denote the amount of rotation from the x-y axis as θ and the tilt angle, i.e. the angle difference between the principal axis and x-y axis as ψ . As such we would have

$$\phi = \psi + \theta \quad (15)$$

where ϕ is the angle of rotation from the principal axis. This relationship is easier to understand by referring to Figure 3.



(a)



(b)

Figure 3 (a): Image showing tilt angle ψ without rotation; (b): Image with an angle of rotation θ In (a), $\phi = \psi$ since θ is zero. In (b), $\phi = \psi + \theta$.

The tilt angle ψ varies with different non-uniform scale factors but the rotational angle θ only depends on the amount of rotation that the image has undergone. Therefore to unrotate the image to the x-y axis, we need to obtain either θ or ψ only since ϕ can be calculated using (14).

We are proposing the method of using a neural network to predict ψ since it is fixed for a particular scale factor of an image and is invariant to rotation which makes it easier for the neural network to be trained rather than using θ which changes for different amounts of rotation. Once the value of the tilt angle ψ is obtained from the neural network, the rotational angle θ can be determined using (15). Next, we can arrive at the moments calculated with respect to the x-y axis, which are also invariant to rotation by using

$$\mu_{pq}^{ur} = \sum_{r=0}^p \sum_{s=0}^q (-1)^{p+q-r} \binom{p}{r} \binom{q}{s} (\cos\theta)^{p-r+s} (\sin\theta)^{q+r-s} (\mu_{p+q-r-s, r+s}) \quad (16)$$

where μ_{pq}^{ur} is the unrotated central moment defined in the x-y axis [11]. By applying these moments into (9) and (13), we are able to obtain features that are invariant to translation, uniform/non-uniform scaling and rotation.

Instead of doing this, we can also use the rotational angle θ to unrotate the image pixel by pixel to the x-y axis and then compute the new moments using (3), (9) and (13) but this method would be erroneous for digital images since there would be some pixels lost while rotating the image. In this paper, we have used the earlier technique since it avoids erroneous values during the unrotation process.

6.0 Neural Network

The new moments that we derived in Section 4.0 are invariant to translation and uniform/non-uniform scaling but not to rotation. In this section, we propose the method of using a neural network to predict the tilt angle ψ , which will be used in the process of obtaining invariance to rotation for the new moments.

A neural network is trained to generate the tilt angle rather than the angle of rotation for the reasons that were mentioned in Section 5.0. A Multilayer Perceptron neural network topology with Backpropagation (MLP-BP) training is used for this purpose [11].

Inputs to the neural network as shown in Figure 4 consist of five features. Out of these, four moments: $\tilde{\eta}_{20}, \tilde{\eta}_{02}, \tilde{\eta}_{30}, \tilde{\eta}_{03}$ (the tilde ‘ \sim ’ sign refers to images that are scaled uniform/non-uniformly) are obtained by applying (3) and (16) to make them invariant to rotation and translation while (5) is used to make them invariant to uniform scaling. The fifth input is the principal axis scale factor i.e. the value of length of the semi minor axis over semi major axis, b/a of the image [9]. These values are invariant to translation, rotation and uniform scaling.

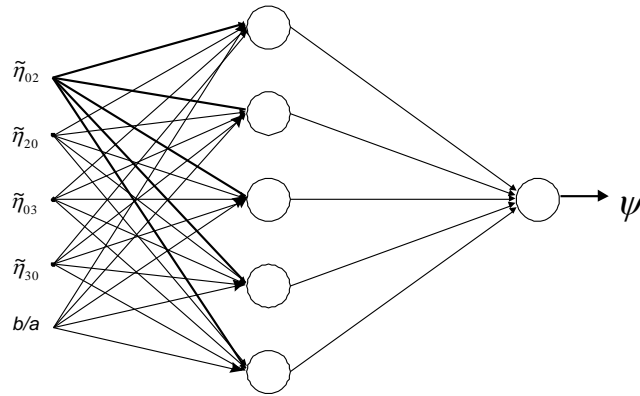


Figure 4: Neural network model to estimate the tilt angle, ψ

Each input is normalised from 0 to 1 to avoid any of them to dominate over the neural network training process. The output tilt angle are also normalised in the same range to speed up the training process. The predicted tilt angle form the neural network is then renormalized to give us the tilt angle in degrees. The topology of the network is fixed at 5:25:1 i.e. 5 input units, 25 hidden units and a single output unit. The training is conducted

until the error in the normalised tilt angle prediction is less than 0.001. The learning rate begins at 0.5 and is reduced with increasing iterations to avoid large oscillations during training. In addition, a momentum value of 0.3 is also used to control the oscillation in the training error.

An important fact to note here is that these input features are not invariant to non-uniform scaling since we need the variance to exist in order to train the neural network to predict the tilt angle, which varies for different non-uniform scale factors for different images. The target value of each image is determined by (14) when the image is not rotated. This gives us the tilt angle ψ when the rotation angle θ is zero.

Once the output of the net is obtained, the angle of rotation from the x-y axis can be determined and hence moments defined in the x-y axis can be calculated. Next, we can apply the methods discussed in the previous sections to arrive at moments that are invariant to translation, rotation and uniform/non-uniform scaling.

7.0 Experimental Study

In section IV, we have successfully shown the invariance of the new moments to non-uniform scaling and translation, both theoretically and through experiments involving English alphabets. In this section, we'll consider experiments with numerals that are rotated, translated and non-uniformly scaled.

The images depicted in Figure 5, 6 and 7 are drawn onto a 128 x 128 grid. Figure 5 shows an example of a class of image numeral 7 that uses non-uniform scaling constants. The tilt

angle is different for each image although the images are not rotated and belong to the same class. Figure 6 shows an example of images of numeral 7 that has been rotated.

This experiment considers the various non-uniformly scaled, rotated and translated images of numeral ‘7’ and the usual regular moment functions are calculated using (3) and (5) for translation and scale invariance. PAMI method [1] is used to obtain invariance to rotation. The entire data set consists of 10 Arabic numerals with each having 81 different combinations of scale factors, α and β . Out of these, 41 different combinations are used for training the neural network and the rest 40 for testing of each class. The training images are not rotated but the test images are rotated with angles 30° , 60° , 150° , 180° , 225° and 300° . Therefore, the test data consists of 240 images for each class.

$\alpha=1.0$ $\beta=1.2$ $\psi=-32.113$	$\alpha=1.0$ $\beta=1.1$ $\psi=-33.878$	$\alpha=1.0$ $\beta=0.95$ $\psi=-36.112$	$\alpha=1.0$ $\beta=0.85$ $\psi=-37.388$	$\alpha=1.2$ $\beta=1.0$ $\psi=-37.587$	$\alpha=1.2$ $\beta=1.15$ $\psi=-35.999$	$\alpha=1.2$ $\beta=1.05$ $\psi=-37.082$	$\alpha=1.2$ $\beta=0.9$ $\psi=-38.532$

Figure 5: Examples of non-uniformly scaled images of numeral 7 used in this paper. The images are shown with their respective tilt angles.

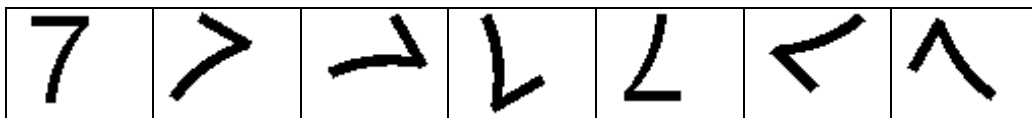


Figure 6: Examples of rotated numeral 7 used in this paper. From left to right, the rotation angles are 0° , 30° , 60° , 150° , 180° , 225° and 300° .

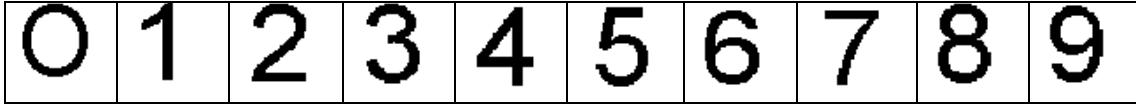


Figure 7: Images (with $\alpha=1.0$, $\beta=1.0$) from the 10 different classes used in the experimental study.

The results of PAMI moment invariants are as tabulated in Table 3 where the sample mean is represented by μ and the sample standard deviation by σ . The high percentage of $|\sigma/\mu|$, which represents the spread of values from their corresponding means, verify that the PAMI features are no longer invariant to non-uniform scaling.

Table 4 deals with experiments conducted to verify that the tilt angle values obtained from the neural network after training are with close agreement with that of the actual values. Numeral 7 is chosen as an example to illustrate this fact. It can be seen from the table that the predicted tilt angle values from the neural network are close to the actual value. The reason for not obtaining the exact values is caused by limited training patterns to the neural network and digitisation errors.

The next table shows the values of the new moments proposed in this paper for examples of numeral 7 that are scaled non-uniformly, rotated and translated. Notice the close agreement of values in each column in Table 5. The reason for not obtaining the exact values is due to the small error surfacing from tilt angle prediction from the neural network. However the small percentage of $|\sigma/\mu|$ validates the proposed method of eliminating the uniform/non-uniform scaling effects in the x and y axis directions while maintaining invariance to rotation and translation.

Table 3: PAMI regular moment functions for non-uniformly scaled and rotated images of numeral 7. α and β correspond to scale change in the x and y axes directions respectively. The angle of rotation is denoted by rot.

Scale factors	η_{20}	η_{02}	η_{21}	η_{12}	η_{30}	η_{03}
Non-Uniformly Scaled Images						
$\alpha=1.0 \beta=1.2$	0.3213	0.1531	0.0010	0.0007	0.0006	0.0009
$\alpha=1.0 \beta=1.10$	0.3788	0.3491	-0.0867	-0.0974	-0.0645	-0.0798
$\alpha=1.0 \beta=0.95$	0.3550	0.2774	0.0052	0.0174	-0.0078	0.0318
$\alpha=1.0 \beta=0.85$	0.3253	0.2487	-0.0033	0.0170	-0.0417	0.0392
$\alpha=1.2 \beta=1.0$	0.1935	0.1284	0.0373	0.0141	0.0285	-0.0209
$\alpha=1.2 \beta=1.15$	0.2905	0.2178	0.0009	0.0029	-0.0096	-0.0059
$\alpha=1.2 \beta=1.05$	0.2352	0.1552	0.0061	-0.0018	0.0093	-0.0160
$\alpha=1.2 \beta=0.90$	0.4235	0.3497	-0.1695	-0.1551	-0.1326	-0.0583
$\alpha=1.2 \beta=0.8$	0.2317	0.1615	-0.0010	-0.0004	-0.0068	-0.0056
$\alpha=1.15 \beta=1.2$	0.2396	0.1627	-0.0073	0.0018	-0.0125	0.0161
$\alpha=1.15 \beta=1.10$	0.2832	0.0200	0.0008	0.0001	0.0001	0.0007
$\alpha=1.15 \beta=0.95$	0.2572	0.2110	-0.0465	-0.0572	-0.0255	-0.0262
$\alpha=1.15 \beta=0.85$	0.2636	0.1431	-0.0003	0.0088	-0.0106	0.0224
$\alpha=1.10 \beta=1.0$	0.2433	0.1235	0.0005	0.0106	-0.0398	0.0302
$\alpha=1.10 \beta=1.15$	0.1542	0.0529	0.0292	0.0112	0.0006	-0.0232
$\alpha=1.10 \beta=1.05$	0.2195	0.1059	0.0021	0.0036	-0.0099	-0.0089
$\alpha=1.10 \beta=0.90$	0.1875	0.0624	0.0046	-0.0019	0.0060	-0.0155
$\alpha=1.10 \beta=0.8$	0.3079	0.1942	-0.0913	-0.1051	-0.0333	-0.0024
$\alpha=1.05 \beta=1.2$	0.1805	0.0707	0.0004	0.0010	-0.0066	-0.0059
$\alpha=1.05 \beta=1.10$	0.1889	0.0687	-0.0051	0.0023	-0.0085	0.0155
$\alpha=1.05 \beta=0.95$	0.2860	0.0447	0.0008	0.0002	0.0002	0.0007
$\alpha=1.05 \beta=0.85$	0.2761	0.2337	-0.0522	-0.0629	-0.0309	-0.0350
$\alpha=0.95 \beta=1.0$	0.2767	0.1660	0.0006	0.0102	-0.0103	0.0238
$\alpha=0.95 \beta=1.15$	0.2550	0.1452	-0.0005	0.0118	-0.0400	0.0316
$\alpha=0.95 \beta=1.05$	0.1592	0.0661	0.0305	0.0105	0.0067	-0.0235
$\alpha=0.95 \beta=0.90$	0.2295	0.1253	0.0018	0.0035	-0.0098	-0.0082
$\alpha=0.95 \beta=0.8$	0.1936	0.0789	0.0049	-0.0018	0.0065	-0.0156
$\alpha=0.9 \beta=1.2$	0.3249	0.2201	-0.1043	-0.1107	-0.0500	-0.0088
Scale factors & rotation angle	Non-Uniformly Scaled and Rotated Images					
$\alpha=0.9 \beta=1.10 \text{ rot}=30^\circ$	0.1873	0.0867	0.0001	0.0008	-0.0065	-0.0058
$\alpha=0.9 \beta=0.95 \text{ rot}=30^\circ$	0.1956	0.0853	-0.0055	0.0021	-0.0091	0.0156
$\alpha=0.9 \beta=0.85 \text{ rot}=60^\circ$	0.2957	0.0873	0.0009	0.0004	0.0003	0.0008
$\alpha=0.85 \beta=1.0 \text{ rot}=60^\circ$	0.3128	0.2761	-0.0640	-0.0745	-0.0422	-0.0512
$\alpha=0.85 \beta=1.15 \text{ rot}=150^\circ$	0.3036	0.2078	0.0022	0.0129	-0.0095	0.0266
$\alpha=0.85 \beta=1.05 \text{ rot}=150^\circ$	0.2791	0.1842	-0.0019	0.0138	-0.0406	0.0343
$\alpha=0.85 \beta=0.90 \text{ rot}=180^\circ$	0.1704	0.0898	0.0327	0.0111	0.0158	-0.0230
$\alpha=0.85 \beta=0.8 \text{ rot}=180^\circ$	0.2503	0.1602	0.0015	0.0033	-0.0097	-0.0072
$\alpha=0.8 \beta=1.2 \text{ rot}=225^\circ$	0.2072	0.1080	0.0054	-0.0017	0.0075	-0.0157
$\alpha=0.8 \beta=1.10 \text{ rot}=225^\circ$	0.3592	0.2682	-0.1278	-0.1244	-0.0804	-0.0245
$\alpha=0.8 \beta=0.95 \text{ rot}=300^\circ$	0.2021	0.1152	-0.0003	0.0004	-0.0066	-0.0056
$\alpha=0.8 \beta=0.85 \text{ rot}=300^\circ$	0.2101	0.1148	-0.0062	0.0020	-0.0103	0.0158
μ	0.2564	0.1539	-0.0151	-0.0155	-0.0171	-0.0047
σ	0.06413	0.08107	0.04421	0.04457	0.02899	0.02578
$ \sigma/\mu \%$	25.016	52.988	292.711	287.268	169.602	553.245

Table 4: Results of tested NN tilt angle output for an example of numeral 7. The error represents the difference between the actual derived theoretical tilt angles and the values that are generated by the trained neural network.

Scale factors	Error	Scale factors	Error
$\alpha=1.0 \beta=1.2$	0.013	$\alpha=1.05 \beta=0.95$	0.000
$\alpha=1.0 \beta=1.10$	0.001	$\alpha=1.05 \beta=0.85$	0.011
$\alpha=1.0 \beta=0.95$	0.003	$\alpha=0.95 \beta=1.0$	0.004
$\alpha=1.0 \beta=0.85$	0.000	$\alpha=0.95 \beta=1.15$	0.012
$\alpha=1.2 \beta=1.0$	0.006	$\alpha=0.95 \beta=1.05$	0.001
$\alpha=1.2 \beta=1.15$	0.003	$\alpha=0.95 \beta=0.90$	0.003
$\alpha=1.2 \beta=1.05$	0.003	$\alpha=0.95 \beta=0.8$	0.002
$\alpha=1.2 \beta=0.90$	0.022	$\alpha=0.9 \beta=1.2$	0.029
$\alpha=1.2 \beta=0.8$	0.019	$\alpha=0.9 \beta=1.10$	0.010
$\alpha=1.15 \beta=1.2$	0.004	$\alpha=0.9 \beta=0.95$	0.004
$\alpha=1.15 \beta=1.10$	0.003	$\alpha=0.9 \beta=0.85$	0.004
$\alpha=1.15 \beta=0.95$	0.008	$\alpha=0.85 \beta=1.0$	0.011
$\alpha=1.15 \beta=0.85$	0.028	$\alpha=0.85 \beta=1.15$	0.043
$\alpha=1.10 \beta=1.0$	0.001	$\alpha=0.85 \beta=1.05$	0.006
$\alpha=1.10 \beta=1.15$	0.004	$\alpha=0.85 \beta=0.90$	0.004
$\alpha=1.10 \beta=1.05$	0.003	$\alpha=0.85 \beta=0.8$	0.003
$\alpha=1.10 \beta=0.90$	0.010	$\alpha=0.8 \beta=1.2$	0.692
$\alpha=1.10 \beta=0.8$	0.029	$\alpha=0.8 \beta=1.10$	0.045
$\alpha=1.05 \beta=1.2$	0.002	$\alpha=0.8 \beta=0.95$	0.012
$\alpha=1.05 \beta=1.10$	0.004	$\alpha=0.8 \beta=0.85$	0.003

Table 6 tabulates the average of results for 240 test images with different values of α , β and rotation angles for each of the 10 numerals used in the study for our newly proposed moments. Again, we are able to observe that the small percentage of $|\sigma/\mu|$ for all the numerals indicates the validity of the proposed new moments.

8.0 Conclusion

The usual regular moment functions, besides invariant to translation and rotation are only invariant to scale change if the scale changes are uniform. In this paper, we have proposed a new technique of obtaining moments for images that are invariant to uniform/non-uniform scaling, translation and rotation. The results of this method for English alphabets and Arabic numerals are shown in Tables 2, 5 and 6. Although exact values are not obtained, the small variance of the results proves that the proposed technique offers a

Table 5: New regular moments as proposed in this paper for non-uniformly scaled and rotated images of 7. α and β correspond to scale change in the x and y axes directions respectively. The angle of rotation is denoted by rot.

Scale factors	η_{20}	η_{11}	η_{21}	η_{12}	η_{30}	η_{03}
Non-Uniformly Scaled Images						
$\alpha=1.0 \beta=1.2$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.0 \beta=1.10$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.0 \beta=0.95$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.0 \beta=0.85$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.2 \beta=1.0$	0.2239	0.0334	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.2 \beta=1.15$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.2 \beta=1.05$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.2 \beta=0.90$	0.2239	0.0332	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.2 \beta=0.8$	0.2239	0.0331	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.15 \beta=1.2$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.15 \beta=1.10$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.15 \beta=0.95$	0.2239	0.0334	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.15 \beta=0.85$	0.2239	0.0331	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.10 \beta=1.0$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.10 \beta=1.15$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.10 \beta=1.05$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.10 \beta=0.90$	0.2239	0.0334	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.10 \beta=0.8$	0.2239	0.0331	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.05 \beta=1.2$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.05 \beta=1.10$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.05 \beta=0.95$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=1.05 \beta=0.85$	0.2239	0.0334	-0.0025	-0.0096	-0.0054	0.0075
$\alpha=0.95 \beta=1.0$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.95 \beta=1.15$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.95 \beta=1.05$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.95 \beta=0.90$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.95 \beta=0.8$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.9 \beta=1.2$	0.2239	0.0332	-0.0025	-0.0097	-0.0054	0.0075
Scale factors; rotation angle Non-Uniformly Scaled and Rotated Images						
$\alpha=0.9 \beta=1.10 \text{ rot}=30^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.9 \beta=0.95 \text{ rot}=30^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.9 \beta=0.85 \text{ rot}=60^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.85 \beta=1.0 \text{ rot}=60^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.85 \beta=1.15 \text{ rot}=150^\circ$	0.2239	0.0332	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.85 \beta=1.05 \text{ rot}=150^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.85 \beta=0.90 \text{ rot}=180^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.85 \beta=0.8 \text{ rot}=180^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.8 \beta=1.2 \text{ rot}=225^\circ$	0.2237	0.0321	-0.0023	-0.0098	-0.0053	0.0072
$\alpha=0.8 \beta=1.10 \text{ rot}=225^\circ$	0.2239	0.0332	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.8 \beta=0.95 \text{ rot}=300^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
$\alpha=0.8 \beta=0.85 \text{ rot}=300^\circ$	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
μ	0.2239	0.0333	-0.0025	-0.0097	-0.0054	0.0075
σ	0.00003	0.00020	0.00004	0.00002	0.00001	0.00005
$ \sigma/\mu \%$	0.013	0.613	1.502	0.230	0.236	0.674

Table 6: Results using new regular moments (shown in terms of average values per numeral) proposed in this paper. The values are for 240 non-uniformly scaled and rotated test images for each of the 10 numerals used in the experimental study.

Numeral	Average values	η_{20}	η_{11}	η_{21}	η_{12}	η_{30}	η_{03}
240 test images of numeral 0	μ	0.28249	-0.00053	-0.00006	0.00001	-0.00104	0.00107
	σ	0.00003	0.00151	0.00000	0.00001	0.00000	0.00000
	$ \sigma/\mu $	0.00009	2.87049	0.07783	1.05051	0.00184	0.00018
240 test images of numeral 1	μ	0.15487	-0.04191	0.02359	0.01131	-0.08135	-0.02378
	σ	0.00480	0.02579	0.00632	0.00744	0.01760	0.00013
	$ \sigma/\mu $	0.03097	0.61542	0.26805	0.65832	0.21634	0.00543
240 test images of numeral 2	μ	0.22389	0.03326	-0.00252	-0.00965	-0.00542	0.00751
	σ	0.00003	0.00020	0.00004	0.00002	0.00001	0.00005
	$ \sigma/\mu $	0.00013	0.00613	0.01502	0.00230	0.00236	0.00674
240 test images of numeral 3	μ	0.20974	-0.00606	-0.00822	-0.02178	-0.03139	0.00275
	σ	0.00007	0.00147	0.00033	0.00007	0.00022	0.00019
	$ \sigma/\mu $	0.00034	-0.24308	-0.04018	-0.00337	-0.00717	0.06790
240 test images of numeral 4	μ	0.14552	0.01341	-0.01554	0.01970	-0.02967	0.01382
	σ	0.00016	0.00395	0.00847	0.00571	0.00723	0.00847
	$ \sigma/\mu $	0.00110	0.29424	0.54504	0.28988	0.24370	0.61307
240 test images of numeral 5	μ	0.19278	-0.01406	-0.01080	-0.00148	0.00632	-0.00038
	σ	0.00013	0.00162	0.00006	0.00010	0.00037	0.00002
	$ \sigma/\mu $	0.00070	0.11535	0.00599	0.06968	0.05905	0.04898
240 test images of numeral 6	μ	0.17705	-0.00897	-0.00653	0.00731	0.00285	0.00117
	σ	0.00004	0.00163	0.00096	0.00253	0.00037	0.00144
	$ \sigma/\mu $	0.00022	0.18189	0.14730	0.34623	0.12981	1.23500
240 test images of numeral 7	μ	0.24821	0.06686	0.06307	-0.04487	0.00639	-0.07088
	σ	0.00251	0.00850	0.01718	0.00949	0.00826	0.01315
	$ \sigma/\mu $	0.01010	0.12720	0.27235	0.21161	1.29207	0.18550
240 test images of numeral 8	μ	0.16607	-0.00154	-0.00599	-0.00043	0.00112	-0.00160
	σ	0.00004	0.00119	0.00001	0.00001	0.00016	0.00000
	$ \sigma/\mu $	0.00022	0.77075	0.00190	0.02921	0.13961	0.00198
240 test images of numeral 9	μ	0.17629	-0.01036	0.00603	-0.00858	-0.00271	-0.00159
	σ	0.00004	0.00085	0.00035	0.00019	0.00032	0.00054
	$ \sigma/\mu $	0.00022	0.08238	0.05795	0.02209	0.11808	0.34159

solution to using regular moment functions for images that are scaled non-uniformly, rotated and translated. Furthermore, it can easily be extended to many different classes and families within the classes.

References

- [1] M.K.Hu. Visual pattern recognition by moment invariants. IRE Transactions on Information Theory, Vol. 8, pp. 179-187, February 1962.
- [2] R.Bamieh and De Figueiredo. A general moments/ attributed-graph method for the three dimensional object recognition from a single image. IEEE Journal of Robotics and Automation, Vol. 2, pp. 31-41, 1986.
- [3] Y.N.Hsu, H.H.Arsenault and G.April. Rotational invariant digital pattern recognition using circular harmonic expansion. Applied Optics, Vol. 21, pp. 4012-4015, 1982.
- [4] T.Wakahara and K.Odaka. Adaptive Normalization of Handwritten Characters Using Global/Local Affine Transformation. IEEE Transactions of Pattern Analysis and Machine Intelligence, Vol. 20, No. 12, Dec. 1998.
- [5] S.W.Lee and J.S.Park. Nonlinear Shape Normalization Methods for the Recognition of Large-Set Handwritten Characters. Pattern Recognition, Vol. 27, pp.895-902,1994.
- [6] F.W.Smith and M.H.Wright. Automatic ship photo interpretation by the method of moments. IEEE Transactions on Computer, Vol. 20, pp. 1089-1094, 1971.
- [7] S.A.Dudani, K.J.Kenneth and R.B.McGhee. Aircraft identification by moment invariants. IEEE Transactions on Computer, Vol. 26, pp. 39-45, 1977.
- [8] R.Palaniappan, P.Raveendran and S.Omatu. Neural Network Classification of Symmetrical and Non-Symmetrical Images Using New Moments with High Noise Tolerance. Special Issue on Invariants for Pattern Recognition and Classification of the International Journal of Pattern Recognition and Artificial Intelligence, Vol. 13, No. 8, pp. 1233-1250, Dec. 1999.

- [9] M.R.Teague. Image analysis via general theory of moments. *Journal of Optical Society of America*, Vol. 70, pp. 920-930, August 1980.
- [10] A.Khotanzad and J.H.Lu. Classification of invariant image representations using a neural network. *IEEE Transactions on Acoustics, Speech and Signal Processing*, Vol. 38, pp. 1028-1038, 1990.
- [11] D.E.Rumelhart and J.L.McCelland. *Parallel Distributed Processing: Exploration in the Microstructure of Cognition*. MIT Press, Cambridge, MA, Vol 1, 1986.
- [12] P.Raveendran and S.Omatu. Neuro-pattern classification of elongated and contracted images. *Information Sciences*, Vol. 3 pp. 209-221, 1995.
- [13] S.O.Belkasim, M.Shridhar and M.Ahmadi. Pattern Recognition with Moment Invariants: A comparative study and new results. *Pattern Recognition*, Vol. 24, pp. 1117-1138, 1991.
- [14] E.B.Elliot. *Algebra of Quantics*. Oxford University Press, New York, 2nd edition, 1913.
- [15] R. Paramesran, P. Ramaswamy and S. Omatu. Regular Moments for Symmetric Images. *IEE Electronics Letter*, Vol. 34, No.15, pp.1481 – 1482, July 1998.
- [16] E.Persoon and K.S.Fu. Shape discrimination using Fourier descriptors. *IEEE Transactions on Systems, Man and Cybernatics*, Vol. 7, pp. 170-179, 1977.