Noise Tolerant Moments for Neural Network Classification

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Abstract

Regular moment invariants face two limitations. First, images with symmetry in the x and/or y directions and symmetry at centroid give zero values for odd orders of central moments. Secondly, they are very sensitive to noise, especially the higher order moments. This paper presents a single solution to solve the symmetrical problem and reduce the noise sensitivity of these moments. The solution involves a new set of moment-based features that uses a reference point other than the image centroid. The reference centre is selected such that the new moment features are invariant to translation, scaling and rotation. The derivation of the new moments and their invariance are shown before experimenting them with some symmetrical alphabets. Next, they are shown to be less sensitive under the presence of Gaussian and random noise as compared to the usual regular moment invariants. Noise corrupted English alphabets are classified with a neural network to further verify the advantage of using the new moment features.

1.0 Introduction

Usual regular moment functions like Hu Moment Invariants (HMI) [2] use central moments to achieve invariance to changes in shift. But the odd orders of these central moments give value of zero for images with symmetry. This poses some problems in pattern representation and classification. Here, images with symmetry are referred to those that have symmetry in the x and/or y directions and symmetry at centroid like a square, equilateral triangle and some of the English characters. This problem is caused by the use of the image centroid in the calculation of the central moments. Since a symmetrical image has pairs of pixels which are equidistant (in opposite directions) from its centroid, the sum of an odd order moment calculation produces a net value of zero. In computing the seven features of the second and third order HMI for images with symmetry in both x and y directions, only two of them produce nonzero values. This causes difficulties in pattern classification, which is especially true under noisy environments where more features are necessary for successful classification since most classifiers are trained with only noiseless images.

Regular moments also express a high degree of sensitivity under the presence of noise and this problem has been the topic of interest for many researchers [1], [7]. These moments are calculated using pixel locations and distance from centroid (or the co-ordinate centre) and because of this, even a few noise pixels are sufficient to affect the calculations. This is since the centroid of most noisy images are closer to signal pixels rather than noise pixels causing an abrupt change in moment values under the presence of noise although the noise grains might be very few in number. This fact is evident since most noisy images have higher signal content than noise content causing the centroid to shift closer to signal pixels rather than noise pixels.

In this paper, both these problems are addressed by formulating a set of new moments that do not give zero values and also reduce the effects of noise. These moments are computed from a centre other than the centroid of the image, the centre being selected to maintain the invariant properties such as translation, scale and rotation. Derivation of the new moments and their invariant properties are shown mathematically. Illustrative computer simulations are included to verify the validity of the proposed method. Comparison of performance of the new moments with HMI using a Multilayer Perceptron with Backpropagation training (MLP-BP) Neural Network (NN) is experimented with characters from Arial and Times New Roman fonts corrupted with Gaussian and random types of noise to further verify to ability of the new moments.

2.0 The Problem of Symmetrical Images

Consider Figure 1. This figure depicts two images that are symmetrical in the x and y axes. The co-ordinate system shown here has its centre at the centroid of the images to eliminate the effects of translation. If p and/or q takes an odd number then μ_{pq} becomes zero. Now, if we were to relocate the co-ordinate system or the centroid of the image to another location, then μ_{pq} becomes nonzero even though p and/or q takes an odd number. The amount of relocation

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is determined by a second order moment equation. The higher orders are not used due to their noise sensitivity.

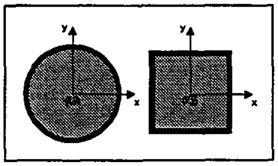

Figure 1: Images with symmetry in the x and y axes

In the following section, we develop the theory and the equations used in selecting this new centre. Selecting a new centre is equivalent to have the original co-ordinate system moved away from the centroid.

3.0 New Moments for Symmetrical Images

The new moment function invariant to translation is defined as:

$$\lambda_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(x - \bar{x} + x_s\right)^p \left(y - \bar{y} + y_s\right)^q f(x,y)\,dx\,dy \quad (1)$$

where the shift terms, x_s and y_s are defined as

$$x_s = c\sqrt{\frac{\mu_{20}}{m_{00}}} \text{ and } y_s = d\sqrt{\frac{\mu_{02}}{m_{00}}} \quad (2)$$

and the shift factor, c and d can take any nonzero value, with $p,q \in N_0$ as order indices, (x,y) are Cartesian co-ordinates; f is a non-negative intensity function with bounded and compact support. The usual central moments, μ_{20} and μ_{02} are calculated from

$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(x - \bar{x}\right)^p \left(y - \bar{y}\right)^q f(x,y)\,dx\,dy \quad (3)$$

For digital images with the limit of $|x|$, $|y| < 1$, (1) can be expressed as

$$\lambda_{pq} = \sum_{-1}^{+1}\sum_{-1}^{+1} \left(x - \bar{x} + x_s\right)^p \left(y - \bar{y} + y_s\right)^q f(x,y) \quad (4)$$

To determine the shift-invariance of λ_{pq}, assume

$$g(x,y) = f(x-x_0, y-y_0) \quad (5)$$

as a shifted version of the given image function, $f(x,y)$. Substituting (5) in (1), it is seen that λ_{pq} is the same for $f(x,y)$ and $g(x,y)$ and hence it is invariant to translation. The new central moments can be normalised to become invariant to scale change by defining,

$$\phi_{pq} = \frac{\lambda_{pq}}{m_{00}^{(p+q+2)/2}} \quad (6)$$

To show the scale invariance of the new moments, let

$$h(x,y) = f\left(\frac{x}{a}, \frac{y}{b}\right) \quad (7)$$

represent a scaled version of the image function, $f(x,y)$. The new central moments, $\tilde{\lambda}_{pq}$ evaluated for $h(x,y)$ is

$$\begin{aligned}\tilde{\lambda}_{pq} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(x - \bar{x} + x_s\right)^p \left(y - \bar{y} + y_s\right)^q f\left(\frac{x}{a}, \frac{y}{b}\right) dx\,dy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(ax - a\bar{x} + ax_s\right)^p \left(by - b\bar{y} + by_s\right)^q abf(x,y)\,dx\,dy \\ &= a^{p+1} b^{q+1} \lambda_{pq} \end{aligned} \quad (8)$$

Combining (6) and (8), the scaled image can be expressed in terms of the original image, ϕ_{pq} as

$$\tilde{\phi}_{pq} = \left(\frac{b}{a}\right)^{\frac{q-p}{2}} \phi_{pq} \quad (9)$$

It is evident from (9) that when the scaling constants $a=b$, i.e. uniform scaling, $\tilde{\phi}_{pq}$ is the same as ϕ_{pq} thus giving us moments which are invariant to scaling. Hu [2], Khotanzad and Lu [3] and Teague [6] and many other authors have all used $a=b$ in solving their problems. We use a similar approach, therefore the new moments are invariant to uniform scaling.

4.0 Experiments with Scaled and Translated Images

Figure 2 shows some of the English alphabets and Arabic numerals from the Arial font with have some symmetrical properties. Characters such as C, D and E are symmetrical along the x axis while characters such as A, M, U, V, W and numeral 8 are symmetrical along the y axis. The numeral 8 might seem to be symmetrical in both the x and y axes but on closer observation, it can be seen that it is symmetrical in the y axis only. Characters H, I, O, X and numeral 0 are symmetrical in both axes. In this experimental study, alphabets O, U, D and Z are considered to illustrate the difference between the new moments and the usual regular moment functions. These images have symmetrical properties along both axes, y-axis, x-axis and symmetry at centroid, respectively. The images are scaled and translated and moments up to the third order are considered. Table 1(a) shows the results using the usual regular moment functions derived by using (1) and (6) with x_s and y_s set to zero i.e. without any shifting and Table 1(b) shows the results of the new moments using (4) and (6) where the value of c and d are fixed arbitrarily at 0.1.

ABCDEHIMNOSTUVWXYZ08

Figure 2: Some symmetrical alphabets and numerals

Notice the images that are symmetrical in the x-axis produce zero values when the q^{th} order moment is odd. Similarly, zero values are obtained when the images are symmetrical in the y-axis and the p^{th} order moment is odd. When the images are symmetrical in both axes then the result is zero whenever p^{th} and/or q^{th} order moments are odd. For images with symmetry in the origin, we find that $p+q$ odd gives zero values. The exact values obtained for symmetrical images within the same class in Table 1(b) validate that the proposed technique is invariant to scale and shift while maintaining a non-zero value for all orders.

5.0 New Moments for Rotated Images

Two techniques are proposed to obtain invariance to rotation for the new moments. The first method involves unrotating the image using the principal axis method described in [2] and from this moments in the principal axis can be obtained, which are invariant to rotation. To perform this, we use the rotation angle ϕ [2] given by

$$\phi = \frac{1}{2}\tan^{-1}\frac{2\mu_{11}}{\mu_{20}-\mu_{02}} \qquad (10)$$

Next the image is unrotated to its principal axis and the new moments are calculated using (1) for continuous images or (4) for digital images. But this method is erroneous for digital images since there would be some pixels lost while performing the unrotation of the image.

To overcome this limitation of the first method, we are proposing another technique to obtain invariance to rotation for the new moments. In the second technique, μ_{pq}^{ur} which is the usual central moment invariant to rotation and translation is used. By expanding (1), a relationship between the usual central moments and the new central moments can be established. In order words, λ_{pq} can be expressed as combinations of x_s, y_s and μ_{pq}, the usual central moment. The general form of the derived relationship for the digital images used in this paper is as given below:

$$\lambda_{pq} = \sum_{-1}^{+1}\sum_{-1}^{+1}\left[\sum_{i=0}^{p}\binom{p}{i}(x-\bar{x})^{p-i}x_s^{i}\sum_{j=0}^{q}\binom{q}{j}(y-\bar{y})^{q-j}y_s^{j}\right] \qquad (11)$$

Combining this relationship with the rotation invariant central moments [3], μ_{pq}^{ur} obtained from

$$\mu_{pq}^{ur} = \sum_{r=0}^{p}\sum_{s=0}^{q}(-1)^{p+q-r}\binom{p}{r}\binom{q}{s}(\cos\phi)^{p-r+s}(\sin\phi)^{q+r-s}(\mu_{p+q-r-s,r+s}) \qquad (12)$$

the new central moments invariant to rotation, λ_{pq}^{ur} can be derived, where the shift terms x_s and y_s are obtained from (2) with μ_{20}^{ur} and μ_{02}^{ur} replacing μ_{20} and μ_{02}, respectively. There are two advantages of using the latter technique. First, it will produce the new moments without any error surfacing from the process of unrotating the image for digital images. Secondly, this method is computationally much faster as compared to the earlier method since it involves only addition operators as can be seen from (11). The moments λ_{pq}^{ur} are inserted into (6) to obtain ϕ_{pq} which is invariant to translation, uniform scaling and rotation. Table 3 shows the values of the new moments for different types of rotated symmetrical images with a 30° angle of rotation using (6), (11) and (12). From this table, it can be seen that the new moments are invariant to rotation. Exact values are not obtained within a class since the images are digital. From the results in Tables 1 and 2, we can conclude that the new moments do not give any zero values for any order and more importantly, they are invariant to translation, uniform scaling and rotation.

6.0 New Moments for Noisy Images

Usual regular moment functions are very sensitive to the presence of noise. This sensitivity is caused by the moments' dependence on the powers of the x-distance and y-distance of each pixel from the centroid (or centre of the co-ordinate system) and since the image pixels are generally located closer to the centroid than the noise speckles, it is evident that the presence of a few noise grains would profoundly affect the values of the moments thus resulting in almost impossible recognition. Although adopting lower order moments will reduce the severity of the problem, the higher order moments represent the finer details of the image and as such are necessary for successful recognition especially under low interclass variance problems. Additional filtering technique can eliminate 'salt and pepper' noise but this method will not work for noise grains of cluster type.

The new moments proposed to solve the symmetrical problem will be used to reduce the effects of noise. In this section, we will use a value of 0.1 for c and d in the shift factor term. For simplicity, we will assume the same value for c and d. Since the new centre is shifted further away from the image centroid, the contribution of the image pixels to the moment calculation is increased. In most cases of noisy images, the signal content is considerably higher than the noise content; as such the contribution of the signal pixels are greatly increased as compared to contribution of noise pixels which only increases slightly.

As a result, the new moments are less sensitive to the presence of noise.

First row of Table 3 shows six different classes of images corrupted with different random noise levels. The noise percentage is calculated based on the image capture area, i.e. the 128 x 128 resolution grid. As an example, 0.5% noise would signify that 81 pixels of noise have been added to the image. Since this paper considers binary images, this means that the values of 81 pixels in the image are changed from 0 to 1 or vice versa.
Table 3 gives the values of HMI moments for these images. The sample mean is denoted by μ, the sample deviation by σ and the percentage spread of moments from their corresponding mean by σ/μ%. From the values of σ/μ%, it can be seen that HMI are very sensitive to noise especially with increasing order. This table also lists the value of the new moments. Comparing the average sum of σ/μ% in Table 3, we can deduce that the new moments are less sensitive under noisy environments as compared to the HMI. The new moments uses a reference centre shifted further away from the image centroid and as a result, the contribution of the signal pixels are greatly increased as compared to the contribution of noise pixels which only increases slightly. This is since for most noisy images, the signal content is much higher than the noise content. As such, the new moments are much less perturbed by the noise pixels. This is also the reason why the new moments are better invariant features than the usual moment invariants for rotated digital images due to loss of signal pixels while performing rotation since loss of signal pixels can be assumed to be a form of noise.

7.0 Experimental Study

All the images for this study are drawn on a 128-resolution image plane and the image plane is mapped onto a square defined by $x\in[-1,+1]$, $y\in[-1,+1]$ before performing any moment calculations. Two experiments using two different data sets of 26 classes, i.e. alphabets from A to Z are used. The first set consists of the Arial font where more than half of the 26 characters have at least one of the symmetrical property. Therefore, this data set can be used as a measure of performance for symmetrical images. Figure 3(a) shows the base set (i.e. without any scaling and rotation) of the alphabets from the Arial font. The second data set consists of alphabets from the Times New Roman font. Figure 3(b) shows the base set of the alphabets from this font. This particular font is chosen since many word processor users use it frequently and also since it consists mostly of non-symmetrical characters. Both these data sets will be used to show that the new moments can perform better that the usual moment invariants for symmetrical and non-symmetrical images as well.

A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	

(a)

A	B	C	D	E	F	G	H	I
J	K	L	M	N	O	P	Q	R
S	T	U	V	W	X	Y	Z	

(b)

Figure 3: (a) Arial (b) Times New Roman alphabets

The data set for each class consists of 13 different scale factors and rotation angles (inclusive of the base image). A comparison is made on the effects of Gaussian and random noise distribution on MLP-BP classification using HMI and the new moments. The probability (adjusted to unit area) of a Gaussian noise pixel occurring at *(x,y)* is given by

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left[-\left(\frac{(x-x_0)^2}{2\sigma_x}+\frac{(y-y_0)^2}{2\sigma_y}\right)\right] \quad (13)$$

where x_0 and y_0 represent the mean of x and y values, σ_x^2 and σ_y^2 represent the x variance and y variance, respectively. Although Gaussian noise type is not common in images, it is experimented since it is common in image processing textbooks. A random function generator is used to obtain the random noise. Each image is subjected to 5 levels of Gaussian and random noise corruption from 0.1% to 0.5% in steps of 0.1. Therefore, the first experiment of using the Arial data set of 26 classes consists of 338 noiseless images, 1690 images with random noise and 1690 images with Gaussian noise. The second experiment of using the Times New Roman data set also consists of a similar amount of images. Since the numerical values of HMI are very small, the logarithms of the absolute values of these functions are used to avoid precision problems. To ensure equal conditions, logarithms of the absolute values are also used for the new proposed moments.

A MLP-BP NN is experimented to further verify the ability of the new moments. The number of hidden units are fixed at 10 and the training error limit is set at 0.001 for all experiments. The new moments are generated with a value of 0.1 for c and d in the shift factor term. The NN classification results for random and Gaussian type noisy images using the usual moment invariants and the new moments are listed in Table 4. From Table 4, we can see that the new moments perform better than HMI for all the noise levels tested for both types of fonts. The new moments are less sensitive to digitisation errors caused by rotation of images, which is shown by the zero noise testing results.

8.0 Conclusion

Odd orders of central moments gives zero value for images with symmetry in the x and/or y directions and symmetry at centroid. These moments are also very sensitive to noise especially the higher order moments. This paper has proposed new moment invariants that solves the symmetrical problem and are more tolerant to noise. These features are computed from a reference centre shifted further away from the centroid of the image. This new centre is selected to maintain the invariant properties. This technique produces nonzero values for all the features of any order and thereby solving the problem caused by symmetrical images for regular moments. Since the new centre is shifted further away from the image centroid, the contribution of both the signal and noise pixels are increased but the contribution of signal pixels is much higher than noise pixels. As such, the new moments are much less perturbed by the noise pixels. This is also the reason why the new moments are better invariant features than HMI for rotated digital images. Extensive results of MLP-BP NN classification for symmetrical and non-symmetrical images using noiseless, random and Gaussian probability noise are presented to validate the proposed method.

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Table 1: Results for scaled and translated symmetrical alphabets using (a) usual regular moment invariants (b) new moments. The scaling constants *a* and *b* correspond to scale change in the x and y directions respectively.

Image	Symmetry type	Scaling factors	η_{02}	η_{20}	η_{11}	η_{12}	η_{21}	η_{03}	η_{30}
O	Both x & y axis	a=b=1.0	1.73e-1	1.89e-1	0.0	0.0	0.0	0.0	0.0
		a=b=2.0	1.73e-1	1.89e-1	0.0	0.0	0.0	0.0	0.0
U	y axis	a=b=1.0	1.41e-1	1.96e-1	0.0	0.0	9.02e-3	8.69e-3	0.0
		a=b=2.0	1.41e-1	1.96e-1	0.0	0.0	9.02e-3	8.69e-3	0.0
D	x axis	a=b=1.0	1.75e-1	2.01e-1	0.0	2.08e-2	0.0	0.0	1.26e-2
		a=b=2.0	1.75e-1	2.01e-1	0.0	2.08e-2	0.0	0.0	1.26e-2
Z	Centroid	a=b=1.0	9.30e-2	2.83e-1	6.25e-2	0.0	0.0	0.0	0.0
		a=b=2.0	9.30e-2	2.83e-1	6.25e-2	0.0	0.0	0.0	0.0

(a)

Image	Symmetry type	Scaling factors	ϕ_{02}	ϕ_{20}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{03}	ϕ_{30}
O	Both x & y axis	a=b=1.0	3.45e-1	3.78e-1	1.81e-1	7.65e-3	7.97e-3	2.20e-2	2.56e-2
		a=b=2.0	3.45e-1	3.78e-1	1.81e-1	7.65e-3	7.97e-3	2.20e-2	2.56e-2
U	y axis	a=b=1.0	2.86e-1	3.91e-1	1.67e-1	6.45e-3	1.65e-2	2.49e-2	2.67e-2
		a=b=2.0	2.86e-1	3.91e-1	1.67e-1	6.45e-3	1.65e-2	2.49e-2	2.67e-2
D	x axis	a=b=1.0	3.51e-1	4.02e-1	1.88e-1	2.87e-2	8.58e-3	2.25e-2	3.97e-2
		a=b=2.0	3.51e-1	4.02e-1	1.88e-1	2.87e-2	8.58e-3	2.25e-2	3.97e-2
Z	Centroid	a=b=1.0	1.86e-1	5.66e-1	1.62e-1	1.12e-2	8.92e-3	1.27e-2	5.49e-2
		a=b=2.0	1.86e-1	5.66e-1	1.62e-1	1.12e-2	8.92e-3	1.27e-2	5.49e-2

(b)

Table 2: New moments proposed in this paper for various types of rotated 30° symmetrical alphabets

Image	Symmetry type	ϕ_{02}	ϕ_{20}	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{03}	ϕ_{30}
O	Symmetry at both x & y axis	3.45e-1	3.78e-1	1.81e-1	7.65e-3	7.97e-3	2.20e-2	2.56e-2
O		3.46e-1	3.79e-1	1.81e-1	7.68e-3	7.92e-3	2.19e-2	2.58e-2
U	Symmetry at y axis	3.51e-1	4.02e-1	1.88e-1	2.87e-2	8.58e-3	2.25e-2	3.97e-2
U		3.51e-1	4.03e-1	1.88e-1	2.87e-2	8.52e-3	2.37e-2	3.97e-2
D	Symmetry at x axis	2.86e-1	3.91e-1	1.67e-1	6.45e-3	1.65e-2	2.49e-2	2.67e-2
D		2.87e-1	3.92e-1	1.68e-1	6.56e-3	1.65e-2	2.50e-2	2.67e-2
Z	Symmetry at centroid	1.86e-1	5.66e-1	1.62e-1	1.12e-2	8.92e-3	1.27e-2	5.49e-2
Z		1.86e-1	5.66e-1	1.62e-1	1.13e-2	8.81e-3	1.27e-2	5.48e-2

Table 3: Results of Hu Moment Invariants and new moments for a noise corrupted image K

	Noise	No noise	0.1%	0.2%	0.3%	0.4%	0.5%	μ	σ	σ/μ %
	Image	K	K	K	K	K	K			
HMI	m_1	2.94e-1	2.97e-1	2.97e-1	3.01e-1	3.03e-1	3.07e-1	3.00e-1	4.71e-3	1.572
	m_2	1.53e-3	1.47e-3	1.56e-3	1.48e-3	1.53e-3	1.56e-3	1.52e-3	3.65e-5	2.400
	m_3	1.45e-3	1.28e-3	1.49e-3	1.58e-3	1.50e-3	1.14e-3	1.41e-3	1.63e-4	11.579
	m_4	4.99e-4	5.95e-4	4.83e-4	5.57e-4	5.64e-4	6.04e-4	5.50e-4	4.95e-5	8.994
	m_5	1.84e-7	2.08e-7	1.66e-7	4.20e-8	4.50e-9	2.05e-7	1.35e-7	8.87e-8	65.688
	m_6	1.25e-5	1.44e-5	1.26e-5	1.07e-5	1.09e-5	1.29e-5	1.23e-5	1.38e-6	11.187
	m_7	3.82e-7	4.76e-7	3.74e-7	5.21e-7	5.18e-7	4.57e-7	4.55e-7	6.43e-8	14.149
	Average of sum for σ/μ%									**16.510**
New Moments	ϕ_{02}	2.55e-1	2.58e-1	2.58e-1	2.62e-1	2.64e-1	2.67e-1	2.61e-1	4.62e-3	1.770
	ϕ_{20}	3.33e-1	3.35e-1	3.37e-1	3.39e-1	3.42e-1	3.46e-1	3.39e-1	4.85e-3	1.433
	ϕ_{11}	1.46e-1	1.47e-1	1.47e-1	1.49e-1	1.50e-1	1.52e-1	1.49e-1	2.37e-3	1.594
	ϕ_{12}	5.83e-3	6.43e-3	6.41e-3	7.61e-3	7.29e-3	6.67e-3	6.71e-3	6.49e-4	9.673
	ϕ_{21}	2.04e-2	2.04e-2	2.07e-2	2.12e-2	2.11e-2	1.97e-2	2.06e-2	5.43e-4	2.638
	ϕ_{03}	1.95e-2	2.17e-2	1.94e-2	1.97e-2	2.01e-2	2.30e-2	2.06e-2	1.43e-3	6.974
	ϕ_{30}	2.93e-2	3.00e-2	2.88e-2	3.07e-2	3.05e-2	3.34e-2	3.05e-2	1.60e-3	5.257
	Average of sum for σ/μ%									**4.191**

Table 4: Results of NN classification for Gaussian and random type noisy images for HMI and the new proposed moments for Arial and Times New Roman fonts

		Times New Roman fonts		Arial fonts	
Noise type	Noise levels	HMI (%)	New moments(%)	HMI (%)	New moments(%)
No noise		94.9	100.0	87.3	99.7
Random	0.1%	72.5	99.4	79.0	89.6
	0.2%	67.2	94.7	71.9	80.5
	0.3%	48.5	89.3	52.7	65.7
	0.4%	39.3	74.9	45.0	56.2
	0.5%	33.1	67.5	40.2	49.4
Gaussian	0.1%	92.9	100.0	83.4	98.5
	0.2%	91.7	99.4	83.4	98.2
	0.3%	88.1	98.8	82.8	96.4
	0.4%	85.8	97.6	83.1	94.3
	0.5%	83.1	98.5	81.9	93.2