against threshold, as shown in Fig. 2, re-confirms the poor performance of one-frame buffered single-layer transmission which falls far below the $PR = 1$ line. Also shown in Fig. 2 are the results of applying M group smoothing (introducing almost the same delay as FSP) which is able to give slightly higher priority to anchor frames; however, the priority ratio is still very close to one.

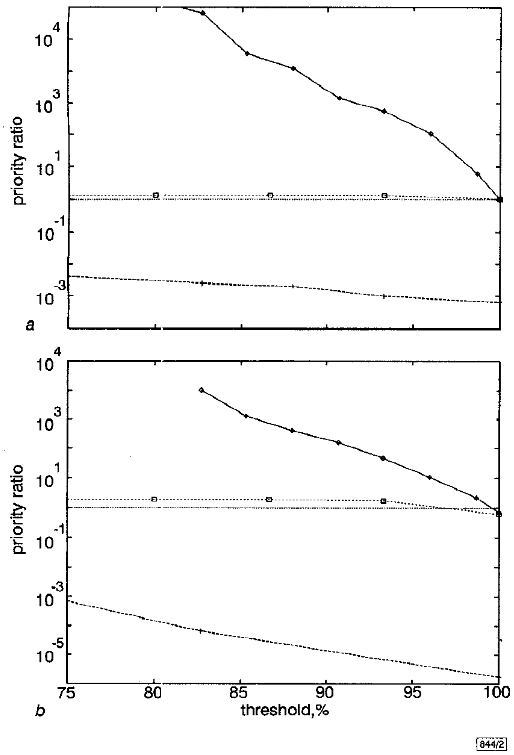

Fig. 2 *Priority ratio against buffer threshold for FSP, one-frame and three-frame smoothing*

a MUSIC
b QUIZ
◇ FSP
+ single: 1 frame
□ single: 3 frames

Fig. 2 clearly shows that the FSP as a loss priority scheme outperforms the conventional method of rate smoothing, where even for strongly aligned sources, B frame cells can be discarded to the direct benefit of anchor frames, resulting in very high PRs and good sensitivity to priority threshold values.

Conclusions: A method for creating two-layer video from a single-layer bit stream has been presented. The benefits of this method are: a continuous B frame stream enabling B frame cell loss to occur in preference to anchor frame cells, even when sources are strongly aligned; there are no data duplication or coding inefficiencies characteristic of two-layer video; a reduction is obtained in the required bandwidths which are no longer as dependent on frame alignments. Even where rate-averaging of each group of M frames may deliver similar advantages regarding required bandwidths and the influence thereof exerted by relative frame alignments, FSP has demonstrated an important edge in being able to better protect anchor frame cells when sources are strongly aligned.

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Regular moments for symmetric images

R. Paramesan, P. Ramaswamy and S. Omatu

Symmetrical images in the x and/or y directions give zero values for odd regular moments, which cause classification problems. It is proposed that regular moments be computed from centres other than the image centroid, where the centre satisfies the translation, scale, rotation, and/or reflection invariance of regular moments.

Introduction: Hu [1] published his classic paper on pattern recognition by deriving a set of regular moment invariants based on algebraic invariants. These regular moments are invariant to changes in scale, shift, rotation and reflection. However, in computing the regular moments for symmetrical images in the x- and/or y- directions such as squares and rectangles, some of the alphabets and numbers produce zero values for odd regular moments. This is because the centroid of the symmetrical image, which is equidistant in the x and/or y directions, is used in the computation of the regular moments to eliminate the effects of translation. In computing the seven features of the second and third order invariant regular moments using the equations shown in [1] for symmetrical images in both the x- and y-directions, only two of them produce non-zero values. This poses some difficulties in pattern classification. We propose that regular moments be computed from centres other than the image centroid, chosen such that the translation, scale, rotation, and/or reflection invariance of regular moments are preserved. This technique produces non-zero values for all regular moments.

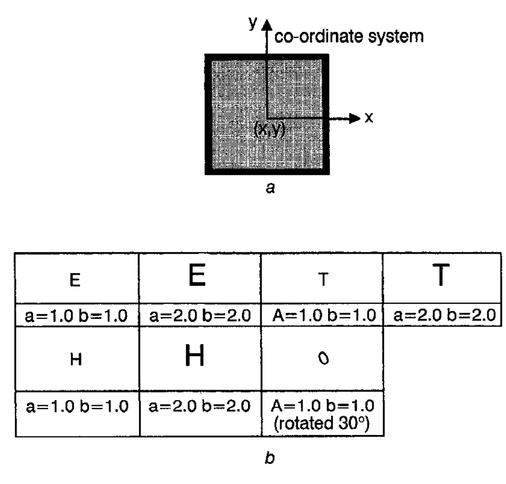

Fig. 1 *Symmetrical image and actual images used in experiments*

a Symmetrical image
b Images used in experiments as shown in Tables 1 and 2

Basic theory: Conventional regular moments are defined as

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)dxdy \quad \text{for } p,q = 0,1,2,... \tag{1}$$

These moments are made invariant to translation and scaling by forming the following invariant [3]:

$$\eta_{pq} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\bar{x})^p(y-\bar{y})^q f(x,y)dxdy}{m_{00}^{(p+q+2)/2}} \quad (2)$$

$\bar{x}$ and $\bar{y}$ are the co-ordinates of the centroid given by

$$\bar{x} = m_{10}/m_{00} \qquad \bar{y} = m_{01}/m_{00} \quad (3)$$

To make them invariant to rotation, Hu combined the regular moments based on the theory of algebraic invariants [1]. Fig. 1 depicts an image that is symmetrical both in the x and y axes. The co-ordinate system shown here has its centre at the centroid of the image to eliminate the effects of translation. If p and/or q take an odd number, then μ_{pq} becomes zero for the image shown in Fig. 1. Now, if we were to relocate the co-ordinate system away from the centroid of the image, then μ_{pq} becomes non-zero even though p and/or q take an odd number. To achieve the usual invariant properties, we move the co-ordinate system determined by the second order moments.

New regular moments: Let us now define a set of regular moments as follows:

$$\tilde{\eta}_{pq} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\bar{x}+\tilde{x})^p(y-\bar{y}+\tilde{y})^q f(x,y)dxdy}{m_{00}^{(p+q+2)/2}} \quad (4)$$

$$\text{for } p,q = 0,1,\ldots \text{ and } \tilde{x} = c\sqrt{\frac{\mu_{20}}{m_{00}}} \text{ and } \tilde{y} = d\sqrt{\frac{\mu_{02}}{m_{00}}} \quad (5)$$

c and d can take any non-zero value. To determine the shift-invariance of $\tilde{\eta}_{pq}$, let $g(x,y) = f(x-x_0, y-y_0)$ be a shifted version of $f(x,y)$. From eqns. 5 and 4, it is readily seen that $\tilde{\eta}_{pq}$ is the same for $f(x,y)$ and $g(x,y)$ and hence is translation invariant. For scaling, let $h(x,y) = f(ax,by)$ and from eqns. 1 – 5, it is evident that the centroid of the scaled image $h(x,y)$, in terms of the original image, $f(x,y)$, is as follows:

$$\bar{\mathbf{x}} = \bar{x}/a \qquad \bar{\mathbf{y}} = \bar{y}/b \qquad \tilde{\mathbf{x}} = \tilde{x}/a \qquad \tilde{\mathbf{y}} = \tilde{y}/b \quad (6)$$

where bold letters are for the scaled image $h(x,y)$. Substituting eqn. 6 in eqn. 4, it is clear that when $\tilde{\eta}_{pq}$ is evaluated for the scaled image $h(x,y)$ and expressed it in terms of the original image, $f(x,y)$, we obtain

$$\tilde{\eta}_{pq} = (b/a)^{(q-p)/2}\eta_{pq} \quad (7)$$

It is evident from eqn. 7 that when the scaling constant $a=b$, $\tilde{\eta}_{pq}$ is the same as η_{pq} thus giving us moments which are invariant to translation and scaling. Hu [1] and Teague [2] have used $a=b$ in solving their problems. To make them invariant to rotation, moments are combined based on the theory of algebraic invariants as shown in [1].

Table 1: Results of regular moments for scaled, rotated and translated symmetrical images using eqn. 2

Image	Scaling constant	Symmetry direction	η_{20}	η_{02}	η_{11}	η_{21}	η_{12}	η_{30}	η_{03}
E	a=1.0 b=1.0	x-	0.090	0.325	0	0	0.015	0.011	0
	a=2.0 b=2.0	x-	0.090	0.325	0	0	0.015	0.011	0
T	a=1.0 b=1.0	y-	0.090	0.428	0	0.047	0	0	0.181
	a=2.0 b=2.0	y-	0.090	0.428	0	0.047	0	0	0.181
H	a=1.0 b=1.0	both	0.171	0.276	0	0	0	0	0
	a=2.0 b=2.0	both	0.171	0.276	0	0	0	0	0
Rotated 30°			$m1$	$m2$	$m3$	$m4$	$m5$	$m6$	
O	a=1.0 b=1.0	both	0.349	1.957	0	0	0	0	

a and b correspond to scaling in x and y directions, respectively

Experiments with scaled, rotated and translated images: Four alphabets, E, T, H and O which have symmetrical properties along the x-axis, y-axis or both axes, are considered to illustrate the difference between conventional regular moments and the new regular moments proposed here. The images are scaled, translated and rotated. The second and third order moment features are used to show the validity of the proposed new regular moments. The results are tabulated in Tables 1 and 2. Table 1 shows the regular moments derived from eqn. 2 and Table 2 shows the results of new regular moments in which the centroid is shifted by a numerical factor as shown in eqn. 5. Note that the images that are symmetrical in the x-axis produce zero values when the qth order moment is odd. Similarly, zero values are obtained when the images are symmetrical in the y-axis and the pth order moment is odd. When the images are symmetrical in both axes then the result is zero whenever the pth and/or qth order moments are odd. The non-zero values and the exact values obtained for symmetrical images of the same class in Table 2 validate that the proposed technique is invariant to scale and shift. Since, in the foregoing analysis, no assumption was made regarding the values the scaling constants a or b may assume, we may allow them to become negative, which treats the appropriate mirror reflections, and means that the proposed regular moments are also invariant to reflection. Rotation invariance can be achieved by combining the regular moments based on the theory of algebraic invariants [1]. Combining the second and third order regular moments, seven rotation invariant features can be derived where the six commonly used, m_1 to m_6 are given in [1]. These values are considered to show the rotation invariant property of the proposed technique. The last row of Table 2 shows the results of the proposed method for rotated image O as compared to the regular moments in the last row of Table 1. In conclusion, the results of the experiments verify the validity of the proposed method.

Table 2: Results of new regular moments for scaled, rotated and translated symmetrical images using eqn. 5

Image	Scaling constant	Symmetry direction	η_{20}	η_{02}	η_{11}	η_{21}	η_{12}	η_{30}	η_{03}
E	a=1.0 b=1.0	x-	0.450	1.627	0.685	0.514	0.992	0.390	2.600
	a=2.0 b=2.0	x-	0.450	1.627	0.685	0.514	0.992	0.390	2.600
T	a=1.0 b=1.0	y-	0.452	2.141	0.786	0.638	1.286	0.379	3.742
	a=2.0 b=2.0	y-	0.452	2.141	0.786	0.638	1.286	0.379	3.742
H	a=1.0 b=1.0	both	0.858	1.380	0.902	0.902	1.144	0.996	2.030
	a=2.0 b=2.0	both	0.858	1.380	0.902	0.902	1.144	0.996	2.030
Rotated 30°				$m1$	$m2$	$m3$	$m4$	$m5$	$m6$
O	a=1.0 b=1.0	both		0.349	0.519	0.805	1.119	2.081	1.379

a and b correspond to scaling in x and y directions, respectively

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Revised connected component algorithm and similarity measure quotient; application in handwritten Bengali character recognition

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A revised connected component algorithm is presented. It has been demonstrated that it is less sensitive to variations in handwritten data with respect to the original algorithm when applied with a novel similarity measure quotient as introduced here. The combined scheme has been successfully applied to recognition of handwritten Bengali characters.