

# Approximate Entropy as an Indicator of Nonlinearity in Self Paced Voluntary Finger Movement EEG

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## Abstract

This study investigates the indications of nonlinear dynamic structures in electroencephalogram signals. The iterative amplitude adjusted surrogate data method along with seven nonlinear test statistics namely the third order autocorrelation, asymmetry due to time reversal, delay vector variance method, correlation dimension, largest Lyapunov exponent, nonlinear prediction error and approximate entropy has been used for analysing the EEG data obtained during self paced voluntary finger-movement. The results have demonstrated that there are clear indications of nonlinearity in the EEG signals. However the rejection of the null hypothesis of nonlinearity rate varied based on different parameter settings demonstrating significance of embedding dimension and time lag parameters for capturing underlying nonlinear dynamics in the signals. Across nonlinear test statistics, the highest degree of nonlinearity was indicated by approximate entropy (APEN) feature regardless of the parameter settings.

Keywords: Electroencephalogram, Nonlinearity, Surrogate data, Approximate entropy

## 1. Introduction

Today, processing and analysis of biological signals such as electroencephalogram (EEG) are actively pursued to improve understanding and diagnosis of pathological conditions; examples of which include epilepsy, dementia, schizophrenia and sleep disorders. Also, there are many research studies on EEG

signals that allow further understanding of brain dynamics of healthy subjects during performance of different cognitive tasks, perceptual tasks, no-task (resting) states and different sleep stages [1, 2, 3]. In addition, recent years have seen many developments involving utilisation of EEG for Brain Computer Interface (BCI) design [4, 5].

Conventional analysis of EEG signals utilise the time and frequency based methods. However the requirements for further characterisation and a better understanding of biological signals have led to an increasing interest in methods adopted from nonlinear dynamics theory [1, 6, 7, 8, 9, 10, 11]. Although signals produced by a biological system seem very unlikely to be linear, their nonlinear nature may not be reflected in recorded signals. In the absence of nonlinear behaviour, it is not favourable to use nonlinear analysis methods as they are more complex and computationally expensive in comparison to their linear counterparts. A requirement exists therefore that, before application of nonlinear analysis methods, the use of such advanced measures should be justified by the properties of the data. For example, nonlinear EEG synchronisation of professional pianists were compared to musically naive subjects during sequential finger movement but without establishing the nonlinear behaviour of the EEG [12].

There are many studies investigating the nonlinearity of EEG signals. The majority of these studies focused on EEG signals recorded from healthy subjects and patients with pathological conditions (i.e. epilepsy, schizophrenia and dementia) as well as signals recorded from patients with sleep disorders during different sleep stages. The general conclusion of these research studies recorded from healthy subjects during resting state hasn't shown any indications of low-dimensional chaos where only weak nonlinearity is observed [1, 13, 14]. On the other hand, there were strong indications of nonlinearity (in some cases associated with low-dimensional chaos) in EEG signals recorded from subjects with pathological conditions compared to EEG signals recorded from healthy subjects [1, 15, 16, 17, 18].

Nonlinear measures such as approximate entropy have been employed to measure the level of anaesthesia [19]. However, only a few handful studies have shown the existence of nonlinearity in EEG signals during self paced movement. Studies in [20, 21] used four nonlinear features, namely correlation dimension, Kolmogorov entropy, nonlinear prediction and largest Lyapunov exponents to analyse the nonlinear dynamic changes in EEG during voluntary self paced movements, which indicated several transients between chaos-like states to almost periodic states. In this study, we perform a comprehensive investigation on the indications of nonlinearity in self paced voluntary finger movement EEG signals using a number of test statistics with the surrogate data method.

The surrogate data method has been used to test for the null hypothesis that the data is generated by a linear stochastic process measured by a memoryless and possibly nonlinear observation function [22]. Testing of the null hypothesis is based upon results generated from seven nonlinear test statistics namely, the third order autocorrelation, asymmetry due to time reversal, delay vector variance method, correlation dimension, largest Lyapunov exponent, nonlinear prediction error and approximate entropy. We have also looked into two different embedding parameter selection methods for estimation of nonlinear test statistics and the significance of embedding parameters on the ability of test statistics for capturing underlying nonlinear structures. The primary aim of this investigation is to demonstrate that the application of nonlinear dynamic measures for characterisation of finger movement EEG signals is justified using approximate entropy as an indicator of nonlinearity.

## 2. EEG Data Set

In this study, we have utilised EEG signals recorded from healthy subjects during an idle (resting) state and during flexion/extension of left index finger. A part of the data set and some of the test statistics have also been utilised in our recent publication investigating the characterisation ability of nonlinear features in comparison to linear features [23].

The EEG data set was recorded from nine right handed subjects (all subjects were male), with ages ranging from 23 to 46. Subject 8 was experienced using a BCI system based on self-paced movement, subjects 3 and 5 had experience in offline BCI experiments and the remaining subjects were naive to BCI use. Signals were acquired using a Guger Technologies g.Bsamp device. EEG signals were recorded over the motor cortex from five bipolar channels located at C3, C1, Cz, C2 and C4, referenced to the right mastoid. Electromyogram (EMG) signals were recorded from the flexors of the left forearm for labeling of movement and non-movement related EEG. All data was sampled at 256 Hz.

Within each run, the subjects were asked to perform self paced flexion/extension of the left index finger whilst a fixation cross was visible on the screen. They were instructed to perform each movement for 5-10 seconds and to rest for a minimum of 10 seconds between movements. As the data was un-cued the number of trials within each run was variable. Each subject performed three runs in a single session. Each run lasted for 610 seconds where the subjects had 5 seconds of pre-waiting and post-waiting periods before and after the fixation cross appeared on the screen for 600 seconds. The timing scheme of a run is illustrated in Figure 1. Instructions were given to concentrate on the fixation cross as much as possible during each run. The EMG signals were observed after each recording session to ensure that the subjects performed flexion/extension of index finger for a sufficient period of time (minimum of 5 seconds) and had sufficient breaks between each movement trial (minimum of 10 seconds).

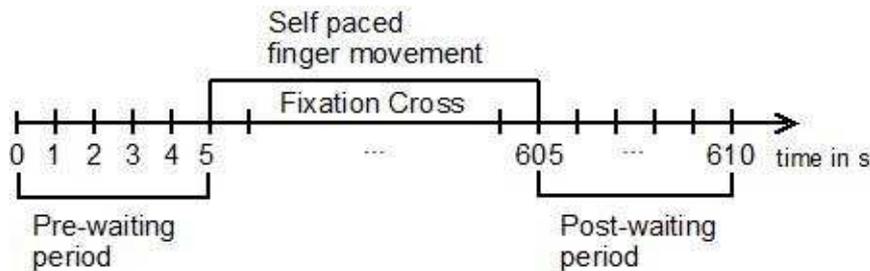


Figure 1: The timing scheme of the experimental paradigm.

## 3. Methods

Most statistical nonlinearity analysis studies utilise the Monte-Carlo approach proposed by Theiler and Prichard [24], which is also referred to as surrogate data method. The surrogate data are the realisations of the null hypothesis that signals are tested against. In the context of nonlinearity analysis, the signals are tested against the null hypothesis of linearity. The idea is to estimate a test statistic from the original data and an ensemble of surrogates that mimic the linear properties of the original data, and test the probability that they come from the same distribution. The null hypothesis is rejected if the test statistic of original data is not from the same probability distribution as surrogates. The essential issues in surrogate data method are

definition of null hypothesis, surrogate data generation method and selection of test statistics.

### 3.1. The Null Hypothesis of Linearity

There are two types of null hypothesis: simple and composite. The simple null hypothesis asserts that the data is generated by a specific linear process. An example of simple null hypothesis would be that the generated data is a random realisation of a specific linear process driven by Gaussian white noise with zero mean and unit variance. Although this hypothesis is straightforward, it is unrealistic - especially for EEG signals, as it is almost impossible to know the specific linear process generating the data. Therefore a more general null hypothesis, referred to as composite null hypothesis would be that the process that generated the data is a member of family of processes. An example of a composite null hypothesis is that the data is generated by a Gaussian white noise with unknown mean and variance.

### 3.2. Surrogate Data Generation

The realisation of composite null hypothesis is achieved by imposing desired linear properties of the original time series on the surrogate data while the rest of the properties are randomised. According to Theiler et. al [25], three linear properties of particular interest are mean, variance and autocorrelation function. The Wiener-Khinchin theorem states that the autocorrelation is equal to the inverse Fourier transform of the power spectrum<sup>1</sup> of corresponding time series [16, 22, 26]. This is related to the fact that linear time series convey all necessary information in the amplitude spectrum while phase spectrum is irrelevant for characterisation of these time series. Thus in the case of linear signals, disruption of phase spectrum does not have any effect on the amplitude distribution of the signal. On the other hand the nonlinear signals have precisely aligned phases and disruption in the phase alignment strongly influences the signal amplitude [16, 27].

Fourier Transform (FT) based surrogates are a straightforward way of realisation of composite null hypothesis that the time series is generated by a linear stochastic process driven by Gaussian white noise. Using this method the surrogates are constrained to preserve the same amplitude spectrum thus having same linear properties (i.e. mean, variance and autocorrelation) as the original data. The FT based surrogate method works well with data which is known to have Gaussian distribution. However in more realistic situations, the time series data does not necessarily follow a Gaussian distribution. In this case, the use of FT based surrogates can lead to false rejection of the null hypothesis. The most general hypothesis that refines deviation from Gaussian distribution is that the times series is generated by a linear stochastic process, driven by Gaussian white noise and followed by memoryless, monotonic and possibly nonlinear observation function  $s(\cdot)$ ,  $s_n = s(x_n)$ . Theiler et. al. [25] proposed Amplitude Adjusted Fourier Transform (AAFT) method for generating the surrogate data following this null hypothesis. With AAFT method, the observation function is used to change signal distribution of original data to follow Gaussian distribution for generation of surrogates and afterwards rescaling the surrogate data back to follow the same distribution as original data. Schreiber and Schmitz [22] demonstrated that the AAFT method can introduce a bias towards a slightly flatter amplitude spectrum, i.e. a white noise spectrum, for short and strongly correlated data. Schreiber and Schmitz proposed the iterative Amplitude Adjusted Fourier Transform (iAAFT)

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<sup>1</sup> i.e. Amplitude spectrum.

method in order to address this problem. It has been shown by Schreiber and Schmitz that the iAAFT method provides an essential improvement over the AAFT method. In this study, we have utilised the iAAFT method to generate the surrogate time series.

### 3.3. Nonlinear Test Statistics

In the literature, the higher order statistics methods and nonlinear dynamics theory methods are widely used for estimating nonlinear test statistics from original and surrogate time series [15, 16, 26]. In this study, we have utilised two measures from the higher order statistics domain, namely the third order autocorrelation and asymmetry due to time reversal and five measures from the nonlinear dynamics theory domain namely the approximate entropy, largest Lyapunov exponents, correlation dimension, nonlinear prediction error and delay vector variance method.

### 3.4. Higher Order Statistics Measures

**Third Order Autocovariance:** The third order autocovariance (C3) is a higher order extension of the autocovariance method that measures the dependence of a time series on the time shifted versions of itself [16]. This measure is given by:

$$C3(\tau) = \frac{1}{N - 2\tau} \sum_{n=2\tau+1}^N (x(n) \cdot x(n - \tau) \cdot x(n - 2\tau)), \quad (1)$$

where  $x(n)$  is the time series,  $N$  is the length of time series and  $\tau$  is the time lag.

**Asymmetry Due to Time Reversal:** Asymmetry due to time reversal (REV) measures the irreversibility of time series, and is an indicator of a strong sign of nonlinearity [16]. This measure is given by:

$$REV(\tau) = \frac{1}{N - 1 - T} \sum_{n=1+\tau}^N (x(n) - x(n - \tau))^3, \quad (2)$$

where  $x(n)$  is the time series,  $N$  is the length of time series and  $\tau$  is the time lag.

### 3.5. Nonlinear Dynamic Measures

**State Space Reconstruction:** The first step in nonlinear dynamic measure estimate is state space reconstruction. At this stage, univariate data is transformed to its trajectory in multidimensional state space. Suppose that a single scalar measure  $\{x(t), t=1, \dots, N\}$  is measured from the system using an observation function  $g(\cdot)$  such that:

$$x(t) = g(s(t)), \quad (3)$$

$$g : M \rightarrow R, \quad (4)$$

$$s(n) \in M \subseteq R^m, \quad (5)$$

where  $s(t)$  stands for the state of system at time  $t$ ,  $M$  is the representation of  $m$  dimensional state space. The single scalar time series,  $x(t)$  will not provide a complete representation of the states of the dynamical system. According to Takens theorem [28], this can be achieved by representing single scalar time series as time lagged versions of itself such that:

$$f : R \rightarrow R^m, \quad (6)$$

$$y_t = f(x(t)) = [x(t), x(t - \tau), \dots, x(t - (m-1)\tau)], \quad (7)$$

where  $\tau$  is time lag,  $m$  is the embedding dimension and  $y_t$  is state vector at time  $t$ .

The selection of the embedding dimension,  $m$ , and time lag,  $\tau$ , parameters are important to achieve a good reconstruction of the time series in state space. In this study, we have used two approaches for the selection of embedding parameters. In the first approach, we utilised conventionally used false nearest neighbors method [27, 29] and first local minimum of mutual information function [27, 29] (MMI&FNN) for selection of these embedding parameters.

In the second approach, we have selected the embedding dimension,  $m$ , and time lag,  $\tau$ , pairs by minimisation of the nonlinear prediction error (GA with NLPE). This method utilises genetic algorithm (GA) for joint estimation of embedding dimension,  $m$ , and time lag,  $\tau$ , parameters. During the estimation process the candidate embedding dimension and time lag pairs are generated and evolved by GA and the quality of reconstruction is assessed with the NLPE measure. The NLPE measure is a locally linear forecasting method that exploits deterministic structures in a time series. This method works by deriving neighbourhood relations from the time series and using these relations to predict future time series points. By using this method, the aim is to obtain an embedding that spreads the data in phase space based on the deterministic dynamic evolution of the system. A more detailed information about this approach can be found in our previous work [30].

**Approximate Entropy Method:** The approximate entropy (APEN) is a measure that quantifies the irregularity of a time series. This was proposed by Pincus [31]. This measure can be estimated as follows:

$$C_i^m = \sum_{j=1}^{Nv} \frac{\Theta(r - \|y_i - y_j\|)}{Nv} \quad (8)$$

where  $Nv$  is the number of vectors in state space,  $r$  is the tolerance of the comparison,  $y_i$  and  $y_j$  are vectors reconstructed in state space  $\|.\|$  represents the Euclidean distance between vectors and  $\Theta(x)$  is the heaviside function such that  $\Theta(x)=1$  if  $x>0$  and  $\Theta(x)=0$  if  $x<0$ . The approximate entropy APEN( $m,r$ ) is obtained by:

$$ApEn(m,r) = \Phi^m(r) - \Phi^{m+1}(r) \quad (9)$$

$$\Phi^m(r) = \frac{1}{N - (m-1)} \sum_{i=1}^{N-(m-1)} \ln[C_i^m(r)] \quad (10)$$

where  $N$  is the length of time series and  $m$  is the embedding dimension.

**Largest Lyapunov Exponent:** Largest Lyapunov exponent (LLE) quantifies the average exponential divergence of nearby trajectories in state space where the sensitive dependence on initial conditions is obtained. In the literature several algorithm has been proposed for the calculation of LLE [11, 27, 29]. In this study, we have used Rosenstein's algorithm [32] where the LLE measure can be estimated as follows:

- For each state space vector  $y_j$  the distance to the nearest neighbor  $y_i$  is calculated:

$$d_j(0) = \|y_i - y_j\|, \quad (11)$$

- Then the two neighboring points are evolved in state space by time  $t$  to calculate the new separation distance:

$$d_j(t) = \|y_{j+t} - y_{i+t}\|, \quad (12)$$

- The largest Lyapunov can be calculated using a least squares fit to the average line defined by:

$$L(t) = \langle \ln d_j(t) \rangle \quad (13)$$

where  $\ln$  is the natural logarithm and  $\langle \cdot \rangle$  denotes the average over all values of  $j$ .

**Correlation Dimension:** Correlation dimension (CD) is a measure of the dimensionality of the space occupied by state vectors [11, 27, 29]. This measure is also referred to as fractal dimension<sup>2</sup>. There are several algorithms for the estimation of CD, in this study we have utilised the Grassberg-Procaccia algorithm [11, 27, 29]. Using this algorithm, the correlation dimension is estimated by first calculating correlation integral,  $C(r)$ , which is defined in (8), over a range of  $r$  values. Then the plot of  $\log C(r)$  versus  $\log r$  should have a linear scaling region whose slope in the limit of small  $r$  and large  $N$ , is the correlation dimension.

$$CD = \lim_{r \rightarrow 0} \lim_{N \rightarrow \infty} \frac{d \ln C(r)}{d \ln r} \quad (14)$$

**Nonlinear Prediction Error:** The nonlinear prediction error (NLPE) is a simple algorithm which exploits the deterministic structure in the time series [27]. This algorithm works by constructing local linear models on a given state space vector.

First, the state vectors  $\{y_t = [x(t), x(t-\tau), \dots, x(t-(m-1)\tau)]\}$  reconstructed from univariate time series,  $\{x(t); t=1, \dots, N\}$  are divided into train,  $Y_{train}$  and test sets,  $Y_{test}$  in which every state vector  $y_t = [x(t), x(t-\tau), \dots, x(t-(m-1)\tau)]$  in the train and test sets has a future sample point,  $x(t+T)$  for  $T$  step ahead prediction. Therefore for every state vector  $y_i$ , with corresponding target  $x(i+T)$  in the test set,  $k$  nearest neighbors from the train set  $\{y_j; j=1, \dots, k\}$ , with corresponding targets  $\{x(j+T); j=1, \dots, k\}$  are grouped together. In order to do the prediction a linear model defined by:

$$x(j+T) = a_0 + \sum_{i=1}^m a_i y_j(i) \quad (15)$$

is fitted to  $k$  state vectors and their target values. The model parameters  $\{a_0, \dots, a_m\}$  are estimated using a recursive least squares algorithm. Following this the prediction error is calculated as  $e = \langle |x(i+T) - \hat{x}(i+T)| \rangle$  where  $\hat{x}(i+T)$  is the predicted sample point and  $x(i+T)$  is the actual sample point. In this study we have set  $T$  to 1 and  $k$  to 1/10 of total number of state vectors in the train set.

**Delay Vector Variance Method:** Delay vector variance (DVV) is a method proposed by Guatama et al. [16] for measuring the unpredictability of a time series in state space and has been applied in a BCI setting

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<sup>2</sup>A fractal dimension is any dimension measurement that allows noninteger values.

[33]. The DVV method involves the following steps:

- The state vectors  $y_t$  are reconstructed in state space from univariate time series,  $x(t)$ , where every vector has a future sample point,  $x(t+T)$ ;
- The mean,  $\mu_d$ , and the standard deviation,  $\sigma_d$ , of all pairwise state vector distances are calculated,  $\|y_i - y_j\|$  where  $i \neq j$ ;
- For each vector,  $y_k$  in state space, sets  $\Omega_k(r_d)$  are created by grouping state space vectors that are closer to  $y_k$  than a certain distance  $r_d$  such that  $\Omega_k(r_d) = \{y_i \mid \|y_k - y_i\| \leq r_d\}$ ;
- The distance  $r_d$  is taken from the interval  $[\max\{0, \mu_d - n_d \sigma_d\}; \mu_d + n_d * \sigma_d]$ . Note that  $n_d$  is a parameter controlling the span over which to perform DVV analysis (set to 4 in this study as suggested by Gautama et al [16]);
- For every state space vector in the set, the variance of the corresponding targets  $\sigma_k^2(r_d)$  is computed. The variance measure is considered valid if the set  $\Omega_k(r_d)$  contains at least 30 state space vectors. The average variance corresponding targets from all sets normalised by the variance of the time series  $\sigma_x^2$  results in the measure of unpredictability  $\sigma^{*2}(r_d)$ :

$$\sigma^{*2}(r_d) = \frac{1}{N} \frac{\sum_{k=1}^N \sigma_k^2(r_d)}{\sigma_x^2} \quad (16)$$

### 3.6. Hypothesis Testing

The null hypothesis of linearity is tested by comparing a nonlinear test statistic, estimated from the original time series and an ensemble of surrogate time series. The null hypothesis is rejected if statistics from the original time series do not come from the same probability distribution as statistics generated from the surrogate time series. Since the distribution of test statistics is not known we have employed a rank-based test as suggested by Theiler and Prichard [24]. A total of  $N_s$  surrogate time series are generated for each of the original time series. The test statistics for the original, to and  $N_s$  surrogate time series,  $\{t_{s,i} \mid i = 1, \dots, N_s\}$  are calculated and the test statistics  $\{t_o, t_{s,i}\}$  are sorted in increasing order, after which the position index  $r$  of  $t_o$  is determined. In this study,  $N_s$  is set to 49.

For hypothesis testing with a significance level of 0.05, a right tailed test is rejected if rank  $r$  of original time series exceeds 47 and a two tailed test is rejected if rank is less than 2 or greater than 48.

One-sided tests are used if the test statistic of the original data deviates from the test statistics of the surrogates only in a specified direction. For DVV statistics, we have performed a right tailed test as it quantifies the predictability of the time series and higher values are expected from original data compared to surrogates. For NLPE statistics, we have performed left tailed test as it quantifies unpredictability of the time series and lower values are expected from original data compared to surrogates. For the rest of the test statistics, we have performed a two-tailed test.

## 4. Results

The null hypothesis of linearity was tested based on each channel for all subjects. The signals from each channel were analysed using a moving window of 256 data points with an overlap of 32

data points. Embedding dimension and time lag pairs for extraction of nonlinear test statistics were estimated individually for each subject. Embedding dimension was set to  $m=4$  for subject 8 and to  $m=5$  for the rest of the subjects; time lag was set to  $\tau = 2$  for subject 8;  $\tau = 3$  for subjects 4, 7;  $\tau = 4$  for subjects 1, 2, 3, 9; and  $\tau = 5$  for subjects 5, 6. The GA with NLPE method led to same embedding dimension and time lag pairs for all subjects except subject 4, where embedding dimension was set to  $m=8$  for subject 4 and to  $m=10$  for rest of the subjects, time lag was set to  $\tau = 1$  for all subjects.

As mentioned in Section 2, the data set was recorded during flexion/extension of left index finger (denoted as 'on' class) and resting states (denoted as 'off' class). Using nonparametric Mann-Whitney test, no significant differences were observed between the rejection rates of on and off classes. Therefore the indications of nonlinearity were investigated based on all EEG segments (without considering the class information) in the corresponding EEG data set.

The rejection of the null hypothesis of linearity rates for EEG segments from different channels using MMI&FNN and GA with NLPE methods are presented in Figures 2 and 3. The graphs are representative of the mean and the standard deviation of rejection rates from each channel averaged over three sessions for each subject.

The graphs in Figure 2 demonstrate that the highest indications of nonlinearity were given by APEN measure for subjects 1-7, DVV and LLE features for subject 8 and DVV, LLE and APEN measures for subject 9. The rest of the test statistics indicated relatively lower rejection rates. Similarly, the results in Figure 3 show that the highest indications of nonlinearity were given by APEN measure for subjects 1-8, DVV and APEN measures for subject 9 and the rest of the features indicated relatively lower rejection rates.

Using statistical Friedman test with a significance level of 0.05, no significant differences were found between the rejection rates of different channels for subjects 1-9 for both cases (MMI & FNN and GA with NLPE)<sup>3</sup>.

Significant differences between rejection rates using the MMI&FNN and GA with NLPE methods for selection of embedding parameters were investigated using a statistical Wilcoxon test with a significance level of 0.05. No significant differences were observed between the rejection rates for all the subjects, except subject 8.

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<sup>3</sup> The statistical tests were performed based on comparing the rejection rates of each feature from different channels and also based on comparing rejection rates of all features from different channels.

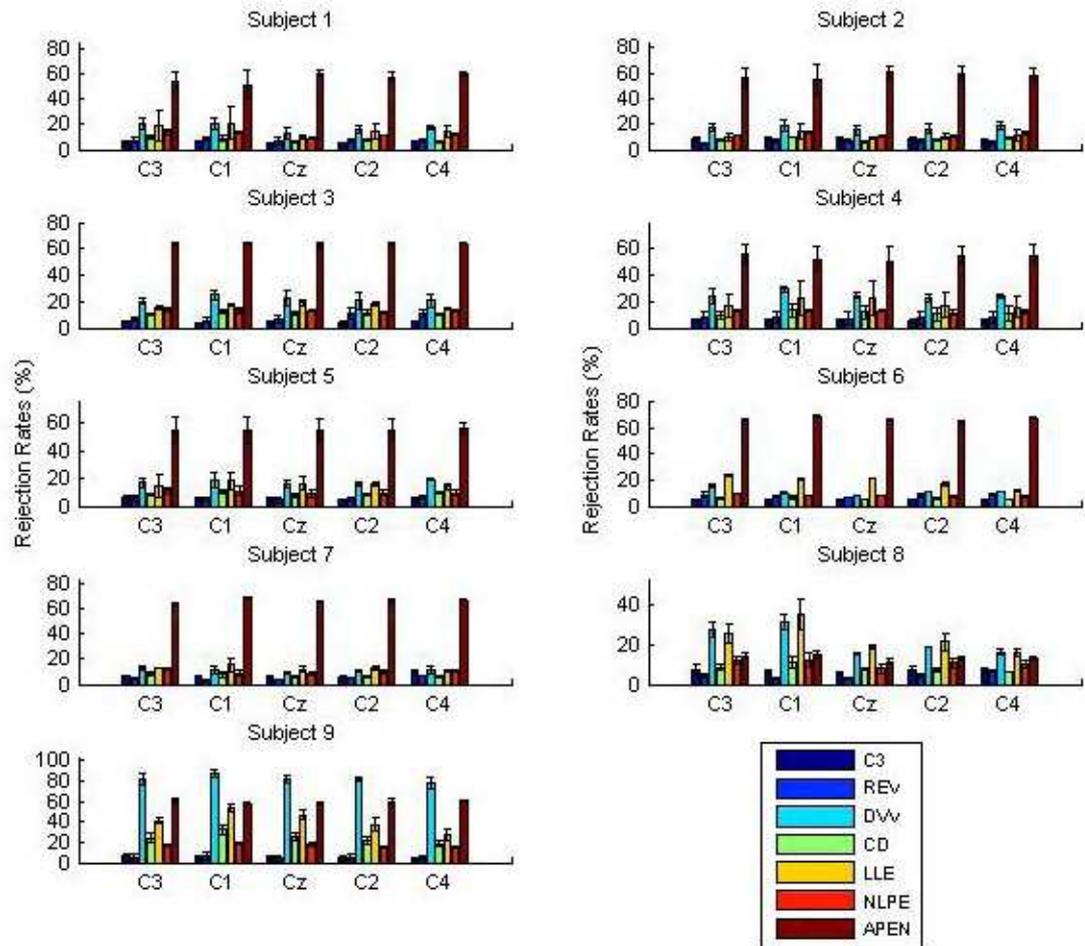


Figure 2: The mean and the standard deviation of the rejection rates of real movement EEG segments grouped by nonlinear test statistics estimated using parameters from MMI&FNN method for each subject and recording site.

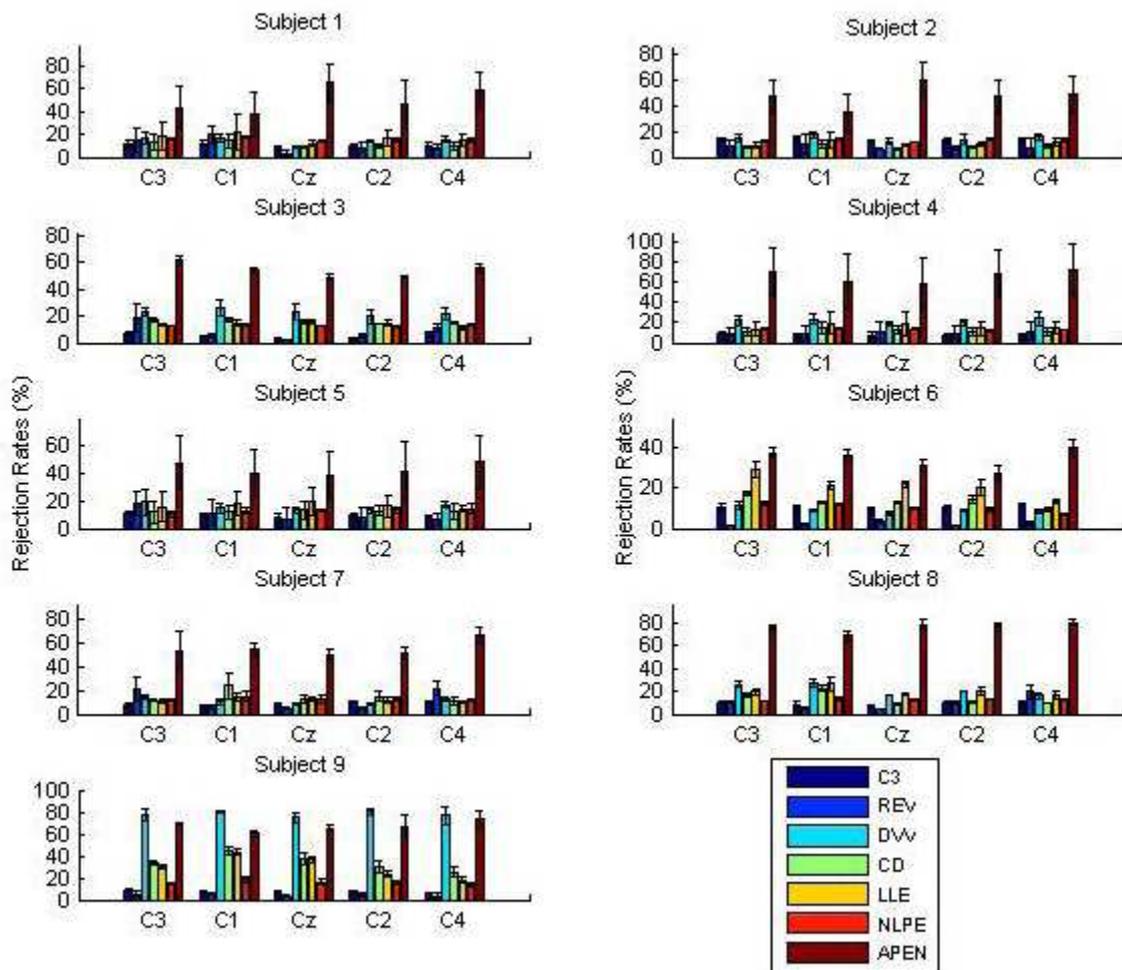


Figure 3: The mean and the standard deviation of the rejection rates of real movement EEG segments grouped by nonlinear test statistics estimated using parameters from GA with NLPE method for each subject and recording site.

## 5. Discussion and Conclusion

In this study, we have investigated the indications of nonlinear structures in self-paced voluntary finger movement EEG. Our main motivation behind the investigation of nonlinear structures in the EEG time series was to justify the applicability of the corresponding nonlinear features for characterisation (i.e. feature extraction) of these signals.

The results have demonstrated that there are clear indications of nonlinearity, with varying rates depending on the test statistics and parameter settings. However, no significant differences were observed between the rejection rates of EEG segments recorded during resting state and exion/extension of left index finger.

Across nonlinear test statistics, APEN feature has consistently indicated highest indications of nonlinearity in all subjects. Moreover the results have illustrated the importance of the selection of embedding dimension and time lag parameters for state space reconstruction. Especially the findings has shown that the selection of optimal embedding parameters is crucial for capturing underlying nonlinear dynamics in the

signals. While the overall results does not prove that the time series are of low-dimensional chaotic nature, the EEG signals were found to be consistent with the hypothesis that there are indications of nonlinear structures in these time series. These findings suggest that the nonlinear test statistics utilised in this study can offer a further characterization and understanding in the context of feature extraction and classification of corresponding self-paced voluntary finger movement EEG signals, which will be useful for a number of applications such as BCI [4, 5], keystroke dynamics [34], biometrics [35] etc.

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